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§1 Editorial

It's a great pleasure to have been invited to act as guest editor for this edition of *The Reasoner*. Fortunately the duties of this position are none too onerous. As ever, it's really Federica Russo to whom we owe our thanks for compiling the articles.

One job I have been given is to conduct an interview which should be of general interest to the readership. As you will see, I have chosen someone who expresses a considerable scepticism as regards formal methods in philosophy. Like me, Brendan Larvor has worked primarily in the philosophy of mathematics, arriving at the similar conclusion that approaches which focus exclusively on logic are missing out on something very important.

In my experience of working and talking with mathematicians, what is noticeable is the huge amount of informal 'chat' going on behind the scenes. I rarely see any desire to apply logical formalisms to the reasoning which is being conducted in these discussions. From 19-23 July I shall be participating in a workshop held in Delphi with some leading mathematicians, where we shall examine each other's views about the possibility of framing as narrative a proof, a mathematical paper, a research programme, a mathematician's career, and a whole movement of research lasting many decades. Judging by the drafts I've seen, these mathematicians see the connection very vividly.

An interesting example of a mathematician who did try to capture more formally something of mathematical reasoning beyond the deductive was George Pólya, who in his 'Mathematics and Plausible Reasoning' worked out qualitative Bayesian reconstructions of pieces of mathematical thinking. I developed some of his ideas in my contribution to *Foundations of Bayesianism* (Corfield and Williamson (eds.), Kluwer, 2001). Imre Lakatos, who you will see features prominently in the interview, praised Pólya for finding commonality between mathematical and scientific reasoning, but disagreed that this was in any way inductive. He criticised Bayesianism for only being relevant to reasoning within a given conceptual framework. However, Pólya had not restricted his Bayesian reconstructions to ordinary induction. He also devoted considerable space to the work of analogy in mathematics, which may be considered to involve conceptual innovation.

All the same, perhaps we shall find that for the foreseeable future that it's on rather less elevated planes of reasoning that formal methods will prove most effective. For the past two years I have been working with a machine learning group in Tübingen. I've been amazed to find how rich a body of mathematics, including Hilbert space theory, is required just to enable a machine to learn how to classify hand-written digits accurately. And this souped-up template matching seems a far cry from any innovative breaking free from a given conceptual framework.

From time to time, a young researcher here will despair at the realization that the field they entered with such high hopes of creating intelligence does not match up to their expectations. Solace is usually found, however, in discovering that machine learning techniques can be pushed to accomplish new tasks by adapting impressive pieces of mathematics. Something which caught my eye recently: if you want to learn how machines can be trained to keep track of airplane identities from their radar traces, then the representation theory of the symmetric groups will stand you in good stead.

David Corfield,
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§2

FEATURES

Interview with Brendan Larvor

Brendan Larvor works at the University of Hertfordshire, where he has been head of the philosophy group for the past four years. Brendan is perhaps best known for his book *Lakatos*, Routledge, 1998.

DC: Having taken a degree at Oxford in mathematics and philosophy, you intended to study for a doctorate with Michael Dummett. What were you going to look at with him, and what changed your mind?

BL: I was going to do intuitionist logic. I had been reading Kant and fretting about infinity. But I was also troubled by the project of pointing at a discourse carried on by serious-minded adults and declaring it to be unintelligible. This is common to the early British empiricists, Kant and the logical positivists. It is particularly unfortunate when the allegedly unintelligible discourse has the intellectual authority of mathematics. Intuitionistic logic requires that you abandon a lot of orthodox mathematics and rewrite most of what remains. I was philosophically curious about mathematics but also about philosophy itself: to what extent is it critical, to what extent is it descriptive?

For personal reasons I deferred my place at Oxford for a year, which I spent at Queens University, Ontario. While I was there, I wrote to David Bloor in Edinburgh. He kindly sent me an alternative philosophy of mathematics reading-list, including Lakatos's *Proofs and Refutations*. Here was a philosopher, Lakatos, studying mathematical activity itself, rather than just looking at formal logic. My vague sense that there is something fishy about mainstream philosophy of mathematics had found a text. So when I met my supervisor, I announced that I intended to write about Lakatos rather than intuitionism.

DC: Did Dummett show much interest in this new line of research?

BL: He was an excellent supervisor, unfailingly courteous and encouraging without stinting on rigour. He was quite happy to supervise a thesis on Lakatos. He was not at all dogmatic, not at all interested in pushing the philosophical positions with which he is associated. To this day, I don't know quite what he thinks about Lakatos, but he did use the phrase 'research programme' in his valedictory lecture.

DC: Can Lakatos still teach us anything today?

BL: That rather depends who 'we' are! Most philosophy of mathematics remains rather narrowly focussed on foundations and ontology, so we philosophers of mathematics still have to learn that there are other philosophical topics. For example, how do mathematicians steer their research? How do they evaluate their own work and that of others? To what extent is mathematics a unity? What is the relation between the content of mathematics and the history of mathematics? In what sense, if any, does the history of mathematics exhibit a rational development? What relationships does mathematics have to other disciplines? I could go on.

Looking beyond the philosophy of mathematics, the 'geometric spirit' (to use Pascal's phrase) is a perennial feature of the modern world. That is to say, the extraordinary success of the mathematical sciences encourages the conviction that any serious problem can be solved by mathematical means. This has had odd and sometimes damaging consequences in economics, social science and philosophy. Lakatos shows that even mathematics itself cannot be understood entirely by mathematical means. What is more, he shows that a critique of the 'geometric spirit' need not fall into an anti-mathematical, please-yourself mystification. In pointing out the limits of metamathematics, he's calling for more rigour, not less.

DC: I bump into you at various conferences. Often we hear someone begin the conference "It's been x years since Lakatos wrote 'Proofs and Refutations'. Now it's time to reform the philosophy of mathematics." Do you feel encouraged for the future of a history/practice-oriented philosophy of mathematics?

BL: Yes and no. There is now a conference circuit for this history-and-practice approach and a rash of recent publications. It is mostly happening in continental Europe, and there are some very impressive young scholars at work. There are definite signs of growth. However, it is all rather eclectic so far. Historians of mathematics are doing fine, but the philosophers of mathematical practice are a mixed bag containing anyone who is dissatisfied with the mainstream. Such a disorderly rabble of malcontents cannot pose a serious challenge to the status quo. We need a more cohesive movement, with an agreed critique of the mainstream.

Part of the trouble is that re-thinking the philosophy of mathematics entails re-thinking the nature of philosophy itself. The idea that philosophy should be largely descriptive is familiar from Wittgenstein and the ordinary language movement. But these moves to valorise practice were still about solving (or dissolving) puzzles, and in any case were hopelessly unhistorical. We want to understand the rationality of the development of mathematics.

Kuhn said that "anyone who believes that history may have deep philosophical import will have to learn to bridge the longstanding divide between the Continental and English-language philosophical traditions." If he is right then we can expect a long road ahead.

DC: In the Anglo-American philosophical world, mathematics has largely been treated in terms of its relationship to logic. You have commented frequently on the limitations of this viewpoint. Do you think that what is being learned in this new philosophy of mathematics should interest the rest of philosophy?

BL: The rest of philosophy is rather diverse and I don't pretend to know what is going on in every corner of it. But formal logic remains at the core of the curriculum in most English-speaking philosophy teaching, and a philosopher's relation to formal logic goes a long way towards deciding what kind of philosopher that person is. Remember that the original analytic/continental split was over the philosophical significance of the (then) new formal logic of Frege and Russell.

The idea that all rationality, be it in thought, speech or deed, can be exhaustively captured in a formal model, runs deep. It is a manifestation of the 'geometric spirit' I mentioned earlier. Insofar as the rest of philosophy is susceptible to this spirit, I think practice-oriented philosophy of mathematics has something to offer. Challenges to the 'geometric spirit' in general and the adequacy of formal logic in particular are too often easy to dismiss as no more than the higher innumeracy of literary intellectuals. Practice-oriented philosophy of mathematics does not suffer that weakness, so I'd like to think it would be of interest to any philosopher, for an afternoon at least.

David Corfield,
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Conceivability, Possibility, and Counterexamples

Many philosophers have been interested in what connection there might be between conceivability and modality, in particular logical modality. In general there are three kinds of accounts that can be developed. Skeptical accounts maintain that there is no connection between conceivability and logical modality. Evidential accounts maintain that conceivability provides evidence of logical possibility. Entailment accounts go further and maintain that conceivability entails logical possibility. Here I want to suggest a line of reasoning in favor of an entailment account.

The line of reasoning I will offer centers around a defense of the claim that it is impossible to give a counterexample to: $C(x, p) \rightarrow \mathbf{L}p$, where $C(x, p)$ stands for p is conceivable to x and $\mathbf{L}p$ stands for it is logically possible that p. Here is a general condition on counterexamples I will employ:

> **Condition (CE):** In order for a subject to give an a priori counterexample to a formula of the form $p \rightarrow q$, two conditions must hold: (a) the subject has to conceive of a scenario S in which p is true, and q is false, (b) S must be possible in some sense of possibility strong enough to ground a counterexample.

Using (CE) we can examine the coherence of giving a counterexample to a conditional linking conceivability with some kind of modality. Let $\mathbf{P}p$ stand for physical possibility, $\mathbf{M}p$ stand for metaphysical possibility, and $\mathbf{L}p$ stand for logical possibility; assume modal pluralism: the set of physically possible worlds is a proper subset of the set of metaphysically possible worlds, which itself is a proper subset of the set of logically possible worlds. Consider the following conditionals:

1. $C(x, p) \rightarrow \mathbf{P}p$
2. $C(x, p) \rightarrow \mathbf{M}p$
3. $C(x, p) \rightarrow \mathbf{L}p$

It is possible to give an a priori counterexample to (1), since if either (2) or (3) is true, a counterexample to (1) will be grounded. For example, if (2) is true, then conceiving of a scenario S in which $C(x, p)$ is true and $\mathbf{P}p$ is false entails that S is metaphysically possible; and since the metaphysical possibility of S is sufficient to ground a counterexample to (1), a subject can reason from (2) and their conceiving of S to the falsity of (1). It is possible to give an a priori counterexample to (2), since if (3) is true, it can ground a counterexample to (2). If (3) is true, then conceiving of a scenario S in which $C(x, p)$ is true and $\mathbf{M}p$ is false entails that S is logically possible; and since the logical possibility of S is sufficient to ground a counterexample to (2), a subject can reason from (3) and their conceiving of S to the falsity of (2).

Following this method, in order to give a counterexample to (3) on the basis of conceiving of a scenario S, conceivability would have to be tied to some kind of possibility strong enough to ground a counterexample to (3). However, if there is no kind of modality wider than logical modality that can ground a counterexample to (3), how is it possible to give a counterexample to $C(x, p) \to \mathbf{L}p$ in virtue of conceiving of a scenario S in which $C(x, p)$ is true, and $\mathbf{L}p$ is false? Consider the following line of reasoning, where S is a putative scenario in which $C(x, p)$ is true and $\mathbf{L}p$ is false, and $\Diamond$ stands for an unspecified kind of modality.

1. $C(x, S)$
2. $C(x, S) \to \Diamond S$
3. $\Diamond S$
4. $\Diamond S \to \neg(C(x, p) \to \mathbf{L}p)$
5. $\neg(C(x, p) \to \mathbf{L}p)$

What kind of modality would $\Diamond$ have to be? On the one hand, if $\Diamond$ is logical possibility, then the argument is self-defeating. For if S is logically possible, then (2) is false; and thus the argument from (1) and (2) to (3) is unsound. On the other hand, if there is no kind of possibility wider than logical possibility that can ground a counterexample, it is impossible to give a counterexample to $C(x, p) \to \mathbf{L}p$. For example, consider epistemic possibility, which can be taken to be wider than logical possibility. Would the epistemic possibility of $C(x, p)$ and $\neg\mathbf{L}p$ show that $C(x, p) \to \mathbf{L}p$ is false? It appears that it would not, since the epistemic possibility of $C(x, p)$ and $\neg\mathbf{L}p$ only shows that for a given subject A, it is compatible with everything A knows that some proposition is conceivable for some person, yet not logically possible. However, that kind of possibility appears to be categorically irrelevant as a kind of modality for grounding a counterexample to $C(x, p) \to \mathbf{L}p$.

Thus, since it is impossible to give a counterexample to $C(x, p) \to \mathbf{L}p$, conceivability entails logical possibility.

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A Counterfactual Account of Essence

Kit Fine (1994, "Essence and Modality", *Philosophical Perspectives*, 8, 1-16) argues that the standard modal account of essence as de re modality is 'fundamentally misguided' (p. 3). We agree with his critique and suggest an alternative counterfactual analysis of essence. As a corollary, our counterfactual account lends support to non-vacuism—the thesis that counterpossibles (i.e., counterfactual conditionals with impossible antecedents) are not always vacuously true.

Explicitly modal accounts of essence are rather recent. Moore (1917-1918) offers an account of essence in terms of entailment: a property F is essential (or *internal*) to x iff '$x = a$' entails 'Fx' (p. 293). Since the emergence of modal logic, essential properties are typically equated with de re necessities. But, as Fine notes, the presumption that there is 'nothing special about the modal character of essentialist claims beyond their being de re' is mistaken (p. 3). While Kripke's wooden table, Tabby, is necessarily a member of the set {Tabby}, it is not essential to Tabby that it be a member of that set. Nor is it essential to Tabby that seven is prime or that it be such that it's either raining or not. The properties: being a member of the set {Tabby}, being such that seven is prime, and being such that it's either raining or not seem irrelevant to the question of what it is to be Tabby. By contrast, the wood of which Tabby is composed seems relevant to Tabby's essence.

In *The Reasoner* 1(1), we offered the following counterfactual explanation of said intuitions: if there hadn't been sets (or if seven hadn't been prime, ...), then Tabby might still have existed. Tabby exists at some (or all) closest impossible worlds at which there are no sets (or numbers, etc.). By contrast, Tabby does not exist at closest worlds where there is no wood.

This sort of explanation requires, for its non-triviality and informativeness, that some counterpossibles be non-trivial and informative, or more specifically, that their truth-values be affected by the truth-values of their consequents. For this reason we take impossible worlds to be *non-deductively closed* sets of sentences. We leave classical logic and one's favored modal logic intact for non-counterpossible modal discourse. We don't have space to discuss closeness of impossible worlds. We simply aim to show that non-trivial counterpossibles make a modal analysis of essences possible. The suggestion: x is essentially F iff if nothing had been F, then x would not have existed.

However, our right-to-left is curious. Thanks to Mike Almeida and Jim Stone for noticing. If Mafia Mike hadn't protected Joey Baddabing, then Joey wouldn't exist. Yet, one might argue, Joey is not essentially protected by Mike. After all, it is not metaphysically necessary that Mike protects him. Worse than this *problem of contingent essences* is the problem of actually *unin-*

stantiated essences. If there were no medical doctors, I wouldn't exist. But I am not a medical doctor. A fortiori, I'm not essentially a medical doctor.

We might modify thus: x is essentially F iff (i) necessarily, if x exists then x is F, and (ii) if nothing had been F, x wouldn't have existed. This modification has the benefit of distinguishing the essential from the necessary while ruling out the essential but contingent (and uninstantiated). On this account an essential property is a metaphysically necessary property that one wouldn't live without. That is, it is a property that x has in every metaphysically possible world in which x exists, and a property such that x does not exist in the closest worlds (possible or impossible) where that property is not instantiated.

We prefer tolerance for contingent essences, and recommend a technical modification to deal with the problem of uninstantiated essences: there being Fs is essential to x iff if there were no Fs then x wouldn't exist. This gets around the problem of actually uninstantiated essences: if there were no doctors, indeed, I would fail to exist. By the above account, there being doctors is (contingently) essential to my existence. But this does not imply that I am a doctor.

The contingent nature of essence is justified by common uses of 'essential'. Consider:

> (1) It is essential to your team's success to advertise your website.
> (2) It is essential to your work to back up your hard drive.

Here 'essential' does not mean what it typically means in recent philosophical literature. (1) doesn't say that there is no world in which you don't advertise your website but your team is successful, and (2) doesn't say that there is no world in which you fail to back up your hard drive but still produce the same work. It's rather something like: holding fixed relevant background conditions, if you don't advertise your website, your team will not be very successful. And holding fixed relevant conditions, if the hard drive is not backed up, your work will eventually suffer. It is natural, then, to understand ordinary essence claims as counterfactuals.

Notice the difference between saying 'x is essentially F' and saying 'F is essential to x'. Your team's success is not essentially such that your website be advertised, but rather said advertising is essential to your team's success. Mafia Mike's protection is essential to Joey's existence, but Joey is not essentially protected by Mike. Doctors are essential to my existence, but I am not essentially a doctor. Whenever x is essentially F, F is essential to x; but sometimes F is essential to x without x being essentially F. This is just what our proposed analyses predict: 'x is necessarily F, and if there were no Fs, x wouldn't exist' obviously entails 'if there were no Fs, x wouldn't exist'.

In conclusion, if one recognizes non-trivial counterpossibles and distinguishes 'is essentially F' from 'F is essential to', one can offer a perfectly general account of 'essential'—an account that captures the philosophical sense and entails the ordinary sense of the term.

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Knowledge, Truth and Justification in Legal Fact Finding

In a previous issue of *The Reasoner* (issue 1(2)), Hock Lai Ho considered 'The Epistemic Basis of Legal Fact-finding'. That article concluded with the suggestion that a criminal conviction would be wrongful where 'the guilty verdict violated K. The court asserted that the defendant was guilty when it did not know that he was guilty (and the court did not know that he was guilty because he was not).' But should we say that a court's decision is wrongful where the court does not know that D is guilty?

Since Gettier's 1963 essay on 'Is Justified True Belief Knowledge?' (*Analysis*, 23, 121-123) there has been reason to doubt that knowledge is a synonym for justified true belief. Williamson (*Knowledge and its Limits*, 2002) has elaborated considerably on the argument that we should consider knowledge to be a separate state from belief, irrespective of whether that belief is justified and/or true. If we distinguish knowledge from belief, and the way in which we use the two terms suggests that we intend to refer to two different mental states, then are we saying that courts should know that a defendant is guilty, or merely hold a justified true belief to that effect?

It is worth deconstructing what we mean when we say 'the court asserted that the defendant was guilty'. This is a statement that operates in a very different way from syntactically similar sentences such as 'the court asserted that the defendant killed the victim', 'the witness asserted that the defendant was guilty' or 'the scientist asserted that DNA evidence is a reliable means of identification'. This is for several reasons, that together combine to make court assertions about truth quite distinctive (although not necessarily unique).

First, an assertion of guilt implies not only a statement of fact, but also combines with it a judgment on the moral value of the defendant's actions (or lack thereof). The effect of that combination is outside the scope of this article, but it should be borne in mind that there are relatively few circumstances in which the court must decide whether the defendant did a particular act.

Secondly, the court, like the journalist in our example, but unlike a witness or a scientist, makes assertions on the basis of evidence presented by others. Our reasoned factual beliefs are based on evidence. We would like to think, as Williamson has suggested, that we classify something as evidence because we know it (p. 189). This may be true for certain categories of evidence, particularly evidence that we derive directly from our own experience. However, the legal fact finding process is one in which people may select, withhold, distort or even fabricate evidence. Even if we set this concern aside and follow the rule that we should tend to believe the testimony of others (Coady, *Testimony*, 1992), we are left with the difficulty that because I know that W asserted E, this does not of itself mean that I know E. The knowledge *W asserted E* is much less useful for my reasoning about facts than would be the knowledge E. There may be circumstances in which I can move from knowing *W asserted E* to knowing E, but as a general rule I can only believe E.

Thirdly, the assertion of the court has practical effect. It is a form of practical reasoning different from that of the reasoning of the journalist. The journalist may influence others, but the court decides the matter (in legal terms) for the defendant (and victim and others).

Fourthly, the assertion of the court closes the matter in a way that the assertion of the journalist or the scientist does not. Once the court has formed its belief, there are only limited circumstances in which that belief can be changed. This is not true for the scientist. While the courts seek to close issues and prevent them from being re-decided, the scientific community seeks almost always to propose that the findings of one piece of work warrant further investigation. The conclusions of science are always provisional. This is true even in an applied science, such as medicine or engineering. If a doctor or engineer asserts a proposed course of action P, and it is then discovered that P was not true, we might say, depending on the circumstances, that the doctor or engineer needs to review her decision. We would be less unlikely to consider P to have been 'wrong' or 'a mistake', and would not say that it was 'wrongful'.

Our approach to whether the courts deal in knowledge or in justified belief may be affected by whether the court is criminal or civil in jurisdiction. The Anglo-American criminal prosecutor must prove her case 'beyond reasonable doubt' while the civil claimant must prove hers 'on the balance of probabilities'. While this allows us to say with a fair degree of certainty that the civil courts do not aspire to establish knowledge, the position is less clear for the criminal courts. However, it is difficult to equate 'reasonable doubt' with something that falls just short of 'absolute certainty', and it seems more likely that criminal verdicts are quantitatively rather than qualitatively different from civil judgments.

One final reason to believe that the courts possess justified beliefs rather than knowledge is that the court system does not possess a unified, persistent consciousness of the sort that we might imagine would be necessary to sustain knowledge. Rather, individual courts appear to form reasoned, justified beliefs for the purpose of deciding instant cases.

On the basis of this analysis, it is not possible to accept the proposition that 'the guilty verdict violated K. The court asserted that the defendant was guilty when it did not know that he was guilty (and the court did not know that he was guilty because he was not).' This is because knowledge is not something that appertains to courts, or to the correct functioning of the courts. Rather, the courts formed truth-indicative justified beliefs, for the purpose of practical reasoning.

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The Principle of Agreement

In a recent issue of *The Reasoner* (volume 1, no. 3, 2007), Stephen Fogdall raises objections to my defense of SPA (the Strong Principle of Agreement). Where $\rho(X, Y)$ is the proportion of members of a set Y that are also in X, I would now formulate SPA as follows:

For $0 \leq r \leq 1$ and for every $\delta > 0$, if U is infinite then:

(1) $\rho_{X,Y}\Big(\rho(X, Y \cap Z) \approx_\delta r \;/\; X, Y, Z \subseteq U \wedge \rho(X, Y) = r\Big) = 1.$

Note that (1) does not say that every triple of subsets of U satisfies the condition that if $\rho(X, Y) = r$ then $\rho(X, Y \cap Z) \approx_\delta r$. Rather, what (1) asserts is that the set of exceptions is a set of measure 0. In *Nomic Probability and the Foundations of Induction* (Oxford, 1990) I defended this by giving some rather weak hand-waving arguments about the relationship between proportions in finite sets and proportions in infinite sets, and this is what Fogdall is objecting to (with some justification). However, I now have a much stronger defense of this and related principles. In an as-yet unpublished paper, "Probable probabilities" (PP), I show that if we make some rather weak assumptions about ρ, then we can prove SPA and related principles. I make three classes of assumptions. Let $\#X$ be the cardinality of a set X. If Y is finite, I assume:

(2) $\rho(X, Y) = \dfrac{\#(X \cap Y)}{\#Y}.$

My second set of assumptions is that the standard axioms for conditional probabilities hold for proportions. These axioms automatically hold for relative frequencies among finite sets, so the assumption is just that they also hold for proportions among infinite sets. I need three further assumptions:

Finite Set Principle: For any set $B, N > 0$, and open formula Φ, $\rho_X\Big(\Phi(X) / X \subseteq B \wedge \#X = N\Big) =$

$\rho_{x_1,\ldots,x_N}\Big(\Phi(\{x_1,\ldots,x_N\})/x_1,\ldots,x_N$
are pairwise distinct $\wedge\ x_1,\ldots,x_N \in B\Big)$.

Projection Principle:
If $0 \le p, q \le 1$ and $(\forall y)(Gy \to \rho_x(Fx/Rxy) \in [p,q])$, then $\rho_{x,y}(Fx/Rxy \wedge Gy) \in [p,q]$.

Crossproduct Principle: If C and D are nonempty then $\rho(A \times B, C \times D) = \rho(A,C) \times \rho(B,D)$.

Note that these three principles are all theorems of elementary set theory when the sets in question are finite. For instance, the crossproduct principle holds for finite sets because $\#(A \times B) = (\#A)(\#B)$, and hence

$$\rho(A \times B, C \times D) = \frac{\#((A \times B) \cap (C \times D))}{\#(C \times D)} = \frac{\#((A \cap C) \times (B \cap D))}{\#(C \times D)} = \frac{\#((A \cap C) \times (B \cap D))}{\#C \times \#D} = \frac{\#(A \cap C)}{\#C} \times \frac{\#(B \cap D)}{\#D} = \rho(A,C) \times \rho(B,D).$$

My assumption is simply that ρ continues to have these algebraic properties even when applied to infinite sets. I take it that this is a fairly conservative set of assumptions. Given these assumptions, we can prove SPA, and a number of related principles. The details are complex (see PP), but the main theorems are as follows.

Given the above assumptions, we can prove:

Law of Large Numbers for Proportions: If B is infinite and $\rho(A/B) = p$ then for every $\epsilon, \delta > 0$, there is an N such that

$$\rho_X\Big(\rho(A/X) \approx_\delta p \;/\; X \subseteq B \wedge \#X \ge N\Big) \ge 1 - \epsilon.$$

Given this law of large numbers we can prove:

Limit Principle for Proportions: Consider a finite set LC of linear constraints on proportions between Boolean compounds of a list of variables $U, X_1, \ldots, X_n$. For any real number r between 0 and 1, and for every $\epsilon, \delta > 0$, if there is an N such that if U is finite and $\#U > N$, then

(2) $\rho_{X_1,\ldots,X_n}\Big(\rho(P,Q) \approx_\delta r \;/\; LC \wedge X_1,\ldots,X_n \subseteq U\Big) \ge 1 - \epsilon$,

then if U is infinite, for every $\delta > 0$:

$$\rho_{X_1,\ldots,X_n}\Big(\rho(P,Q) \approx_\delta r \;/\; LC \wedge X_1,\ldots,X_n \subseteq U\Big) = 1.$$

The finite principle of agreement (FPA) is a straightforward theorem of set theory (proven in PP)):

Finite Principle of Agreement: For $0 \le r \le 1$ and for every $\epsilon, \delta > 0$, there is an N such that if U is finite and $\#U > N$, then:

$$\rho_{X,Y}\Big(\rho(X, Y \cap Z) \approx_\delta r \;/\; X, Y, Z \subseteq U \wedge \rho(X,Y) = r\Big) \ge 1 - \epsilon.$$

SPA (equation 1) follows from FPA via the Limit Principle.

SPA is just one instance of a large class of theorems about "probable probabilities". The following theorem about proportions in finite sets is a theorem of finite combinatorial mathematics:

Probable Probabilities Theorem:
Let $U, X_1, \ldots, X_n$ be a set of variables ranging over sets, and consider a finite set LC of linear constraints on proportions between Boolean compounds of those variables. If LC is consistent with the probability calculus, then for any pair of Boolean compounds P, Q of $U, X_1, \ldots, X_n$ there is a real number r between 0 and 1 such that for every $\epsilon, \delta > 0$, there is an N such that if U is finite and $\#U > N$, then

(2) $\rho_{X_1,\ldots,X_n}\Big(\rho(P,Q) \approx_\delta r \;/\; LC \wedge X_1,\ldots,X_n \subseteq U\Big) \ge 1 - \epsilon$.

Nomic probabilities are defined to be proportions among infinite sets of physically possible objects. Letting "$X \preccurlyeq Y$" mean "X is a subproperty of Y" (i.e., being an X nomically implies being a Y, or equivalently, the set of physically possible X's is a subset of the set of physically possible Y's), and given the limit principle for proportions, the Probable Probabilities Theorem entails:

Expectable Probabilities Principle:
Let $U, X_1, \ldots X_n$ be a set of variables ranging over properties and relations, and consider a finite set LC of linear constraints on probabilities between truth-functional compounds of those variables. If LC is consistent with the probability calculus, then for any pair of truth-functional compounds P, Q of $U, X_1, \ldots, X_n$ there is a real number r between 0 and 1 such that for every $\delta > 0$,

$$\mathrm{prob}_{X_1,\ldots,X_n}\Big(\mathrm{prob}(P,Q) \approx_\delta r \;/\; LC \wedge X_1,\ldots,X_n \preccurlyeq U\Big) = 1.$$

I will express this result more simply by saying that, given the constraints LC, the expectable value of $\mathrm{prob}(P/Q) = r$. It is of interest that this approach also allows us to solve the problem of identifying the Y-function[1]. If we define:

$$Y(r, s|a) = \frac{rs(1-a)}{a(1-r-s) + rs}$$

we can then prove:

Y-Principle:
If $B, C \preccurlyeq U$, $\mathrm{prob}(A/B) = r$, $\mathrm{prob}(A/C) = s$, and $\mathrm{prob}(A/U) = a$, then the expectable value of $\mathrm{prob}(A/B \wedge C) = Y(r, s|a)$.

[1] See my (2007: "The Y-function", in William Harper and Gregory Wheeler (eds.), *Probability and Inference: Essays in Honour of Henry E. Kyburg Jr.*, College Publications)

Of course, to get these results we are still making assumptions about the behavior of ρ in infinite sets, but I take it that the assumptions required are quite abstemious.[2]

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A Note on Kripke's Puzzle about Belief

In his recent contribution to this gazette, Jesse Steinberg (2007: "Pierre may be ignorant, but he's not irrational", *The Reasoner* 1(3): 2-3) discusses the following rendition of Kripke's puzzle, due to Sosa:

1. Pierre is rational. (assumption)
2. Pierre, on reflection, assents to "Londres est jolie". (assumption)
3. "London is pretty" is a translation of "Londres est jolie". (assumption)
4. Pierre, on reflection, assents to "London is not pretty". (assumption)
5. Pierre believes that London is pretty. (2, 3, D)
6. Pierre believes that London is not pretty. (4, D)
7. Pierre believes that London is pretty and Pierre believes that London is not pretty. (5, 6, I)
8. If Pierre believes that London is pretty and Pierre believes that London is not pretty, then Pierre has contradictory beliefs. (analytic?)
9. Pierre has contradictory beliefs. (7, 8, MP)
10. If Pierre has contradictory beliefs, then Pierre is not rational. (analytic?)
11. Pierre is not rational. (9, 10, MP)

Steinberg aims to provide a solution to this purported paradox, by denying premise (10). According to him, the mere fact that an agent has contradictory beliefs is not a sufficient condition to count that agent as irrational. Steinberg supports this claim with the example of Tim, a student who rightfully believes that the form of a certain argument (*modus ponens*) is valid, but then fails to recognise the validity of a different argument of the same form. Now, Steinberg asks: "Would we consider Tim to be irrational? Tim is surely not astute. One might be tempted to call him obtuse, but he is certainly not irrational. [...] What would make Tim irrational is his believing a contradiction, his being aware that he believes that contradiction, and his obstinacy in continuing to believe the contradiction even in the face of this awareness." (2007: 2-3)

The intended moral of this example is that an agent *A* might have contradictory beliefs and yet continue to be rational. According to Steinberg that may happen if (i) *A* has two contradictory beliefs, (ii) *A* is ignorant of having beliefs that are contradictory, and (iii) *A* is disposed, or able, to revise his belief system upon becoming aware of the contradiction.

My intention here is not to criticise this suggestion. Though some further qualification may be needed (e.g., to the effect that *A*'s ignorance must not be due to any obvious fault in *A*'s reasoning capacities), the idea that rational agents might have contradictory beliefs is not so unintuitive, and has in fact received compelling support from various philosophers (e.g., Dummett 1973: *Frege: Philosophy of Language*, London: Duckworth).

What I do intend to argue, in effect, is that this idea is not necessary—and, indeed, not even adequate—for the task of solving Kripke's puzzle. To see this, let us follow Steinberg's suggestion and grant that Pierre is ignorant, but not irrational. In other words, Pierre has contradictory beliefs, but is unaware of the contradiction, and therefore (at least potentially) rational. Now the question is: What is Pierre actually ignorant of? Presumably, it is the fact that there is only one city which he calls 'Londres' in French and 'London' in English. Obviously, there is no irrationality involved in this cognitive shortcoming: one may fully rationally employ different idiolects, without having to know all the correspondences (i.e., standard translations) between those idiolects. But let us imagine next that Pierre somehow learns that the names 'Londres' and 'London' in fact denote the same city and reports his discovery (of an *a posteriori* piece of knowledge) in French: "Incroyable! Après tout, Londres est London!". Now, applying Kripke's translation principle, together with his remark that the translation of 'Londres' as 'London' "[i]s a standard one, learnt by students together with other standard translations of French into English" (Kripke 1979: "A Puzzle about Belief", in *Meaning and Use*, p. 128), we should have no qualms about translating Pierre's words into English as: "Incredible! After all, London is London!" Yet, it seems pretty clear that this translation would be inadequate, since it would have Pierre foolishly rejoicing in the discovery of a trivial a priori identity statement—which is clearly not what his French utterance reports.

This suggests to me that in cases like Pierre's—i.e., when the speaker is unaware of certain facts about translation between idiolects—our own translation of the speaker's utterances should be guided, and appropriately constrained, not only by what Kripke calls the translation principle, but also by a principle of charity which implies, among other things, that we should aim

[2] This work was supported by NSF grant no. IIS-0412791. Thanks to Stephen Fogdall for his comments.

at preserving both the truth-value of the speaker's assertions, and their cognitive content. (There are other imaginary examples that support this diagnosis. Suppose, for instance, that Pierre is blindfoldedly taken to a (fictional) *Quartier Français* in London. Without knowing where he is, Pierre sees a placard that reads: "*Bienvenus à Londres!*". As he enjoys the neighbourhood a lot, he says to himself: "*Allors, ça c'est Londres! C'est une ville merveilleuse! J'aimerais bien vivre ici! Quel dommage que j'habite à London!*" If we were to translate Pierre's assertions in the standard way, we would get the following result: "*So, this is London! What a wonderful city! I'd love to live here! Too bad I live in London!*" It goes without saying that this is not an adequate translation of Pierre's words, since it has him contradict himself, which is not what he's doing in French.)

The preceding remarks, if correct, suggest a different way of tackling Kripke's puzzle, which enables us to block the apparent paradox before it even gets off the ground. If the translation of 'Londres' (in Pierre's idiolect) as 'London' is unwarranted, as I have argued, then premise (3) of the argument is false (N.B. as applied to Pierre's idiolect, not as a rule of standard translation). And this, in turn, blocks the derivation of line (5). Not only is Pierre "merely ignorant and not some sort of bizarre irrational being", as Steinberg argues. He is not even committed to any contradiction in the first place.

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§3 News

Calls for Papers

Introducing ...

The Reasoner would like to publish very short introductions to key terms, people and texts in logic and reasoning. Selected pieces will also be published in a book "Key Terms in Logic" by Continuum. If you would like to contribute, please contact TheReasoner@kent.ac.uk

Theoretical Computer Science: Special issue in honour of Jean-Yves Girard on the occasion of his 60th birthday year, deadline 30 September.

Connection Science Journal: Special issue on Social Learning in Embodied Agents, alberto.acerbi@istc.cnr.it, deadline: 30 October 2007.

Special Issue of Foundations of Science: Mathematics and Argumentation, deadline 1 November 2007.

Erkenntnis: Special Issue on Conditionals and Ranking Functions, franz.huber@uni-konstanz.de, Deadline for submissions: May 31, 2008.

§4 Events

LORI: Logic, Rationality and Interaction, Beijing, 5-9 August 2007.

TANCL'07: Algebraic and topological methods in non-classical logics III, 5-9 August 2007, Oxford.

Workshop: Construction and properties of Bayesian nonparametric regression models, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, August 6-10 2007.

LMPS: 13th International Congress of Logic, Methodology and Philosophy of Science, Beijing, 9-15 August 2007.

IJCNN2007: 2007 International Joint Conference on Neural Networks, Orlando, Florida, August 12-17, 2007.

Uni-Log: 2nd World Congress and School on Universal Logic, Xi'an, 16-19 August 2007.

C&O:RR-2007: The Third International Workshop on Contexts and Ontologies: Representation and Reasoning, August 21, 2007, CONTEXT Workshop Program, Roskilde University, Denmark.

LSFA'07: Second Workshop on Logical and Semantic Frameworks, with Applications, August 28th, 2007, Ouro Preto, Minas Gerais, Brazil.

ASAI 2007: IX Argentine Symposium on Artificial Intelligence Mar del Plata, Argentina, August 27-28, 2007.

Progic 2007

The Third Workshop on Combining Probability and Logic, University of Kent, 5-7 September 2007.

BLC 2007: British Logic Colloquium, London, September 6-8, 2007.

IDA 2007: The 7th International Symposium on Intelligent Data Analysis, Ljubljana, Slovenia, September 6-8, 2007.

Dynamics of Knowledge and Belief: Workshop at KI-2007, 30th Annual German Conference on Artificial Intelligence, Osnabrück, 10 September 2007.

CSL 2007: Computer Science Logic, 11-15 September, 2007, Lausanne (CH).

International Conference on Normative Concepts: Zurich University, 21 - 22 September 2007.

AIPL-07: Workshop on Artificial Intelligence Planning and Learning, Providence, Rhode Island, September 22, 2007, organized in conjunction with the International Conference on Automated Planning and Scheduling (ICAPS-07).

SymCon'07: The Seventh International Workshop on Symmetry and Constraint Satisfroblems, Providence, RI, USA, September 23rd 2007.

ICAPS 2007: Workshop on Planning in Games, Providence, Rhode Island, USA, September, 23, 2007.

Spring Bayes 2007: The 4th annual meeting of Australasian Society for Bayesian Analysis (ASBA) will take place in Coolangatta, 26-28 September, 2007.

TBILISI: The Seventh International TBILISI Symposium on Language, Logic and Computation, 1-5 October 2007.

Reason, Intuition, Objects: The Epistemology and Ontology of Logic, Buffalo, 13 October 2007.

LPAR 2007: Logic for Programming, Artificial Intelligence and Reasoning, Yerevan, Armenia, 15th-19th October 2007.

Women in Machine Learning: Orlando, Florida, October 17, 2007.

Case Studies of Bayesian Statistics: The Ninth Workshop on Case Studies of Bayesian Statistics, October 19th and 20th, 2007 at Carnegie Mellon University, Pittsburgh, PA.

MWPMW 8: Eighth annual Midwest PhilMath Workshop, to be held at Notre Dame, October 27th and October 28th.

ECSQARU'07: Ninth European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty, October 31, November 1-2 2007, Hammamet, Tunisia.

Conference and Call for Papers: 18th Novembertagung on the history, philosophy and didactics of mathematics, Bonn, Germany, November 1-4, 2007.

International Conference on Infinity in Logic and Computation: 3-5 November 2007, University of Cape Town, South Africa.

Workshop: 3rd Workshop on Uncertainty Reasoning for the Semantic Web, Busan, Korea, November 12, 2007.

EPSA07: 1st Conference of the European Philosophy of Science Association, Madrid, 15-17 November 2007.

Graduate Conference: 1st Cambridge Graduate Conference on the Philosophy of Logic and Mathematics, 19th-20th January 2008 St. John's College, Cambridge.

Reduction and the Special Sciences: Tilburg Center for Logic and Philosophy of Science, 10-12 April 2008.

Causation: 1500-2000: King's Manor, University of York, 25-27 March 2008.

Workshop: XVIII Inter-University Workshop on Philosophy and Cognitive Science, Madrid, April 22nd-24th 2008, luis.fernandez@filos.ucm.es.

ISBA08: 9th World Meeting of the International Society for Bayesian Analysis (ISBA), Hamilton Island, Australia, 21st-25th July 2008.

COMMA'08: Second International Conference on Computational Models of Argument Toulouse, France, 28-30 May 2008.

First Formal Epistemology Festival: Conditionals and Ranking Functions, Konstanz, July 28-30, 2008, Deadline for submissions: February 29, 2008.

Soft Methods for Probability and Statistics: 4th International Conference, Toulouse, France, September 8-10, 2008.

Valencia Meetings: Valencia / ISBA Ninth World Meeting on Bayesian Statistics, Spain, June 2010.

§5
Jobs

Postdoctoral jobs in AI: Centre for Artificial Intelligence, Universidade Nova de Lisboa, Portugal, 5-year contracts, deadline, 31 August 07, contact: lmp@di.fct.unl.pt.

Senior Research Fellowships: All Souls College, University of Oxford, deadline 10th September 2007.

8 - 12 Postdoctoral Researcher / University Researcher positions: Helsinki Collegium for Advanced Studies, University of Helsinki, deadline 12 September 2007.

2-year Postdoc: Konstanz University, Germany. The Emmy Noether junior research group Formal Epistemology, two year postdoctoral research position in Philosophy, on the project 'Belief and Its Revision', deadline November 1, 2007.

§6
Courses and Studentships

Courses

Research Master in Language, Cognition, Action, and Mind Studies: The Institute for Logic, Cognition, Language, and Information of the University of the Basque Country (Donostia-San Sebastian).

Summerschool: 19th European Summer School in Logic, Language and Information, Dublin, Ireland, Aug. 6-17.

Logic Summer School: Italian Association of Logic and its Applications (AILA), Italian Society for Logic and Philosophy of Science (SILFS), Palazzo Feltrinelli, Gargnano, Italy, 26 August - 1 September 2007.

Formal Methods in Philosophy and Linguistics: Summer School 2007, August 19 - August 31, Tartu, Estonia.

SECEVitA 2007: Summer School in Artificial Life and Evolutionary Computing, 31 August – 4 September 2007, Baia Samuele, Ragusa, Italy

Discrete Choice Modelling: Centre for Transport Studies at Imperial College London, in London on 28th to 30th November 2007.

Second Indian Winter School on Logic: January 14-26, 2008, IIT Kanpur.

Studentships

BSPS Doctoral Scholarship in Philosophy of Science, closing date 1st August 2007.

PhD Studentship in Statistics: Fully funded PhD Studentship in Statistics, Umea University, Northern Sweden, deadline August 23 2007.

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