Far-from-equilibrium dynamics of systems with conservation laws

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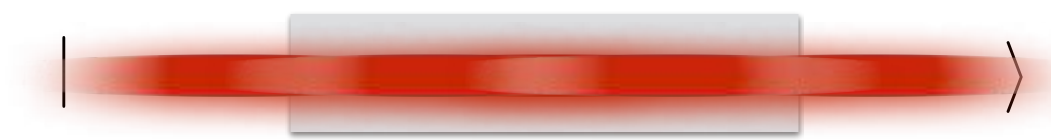
Condensed Matter Physics in All the Cities: Online 2020

Quantum thermalization

Investigate whether/how closed quantum many-body systems thermalize:

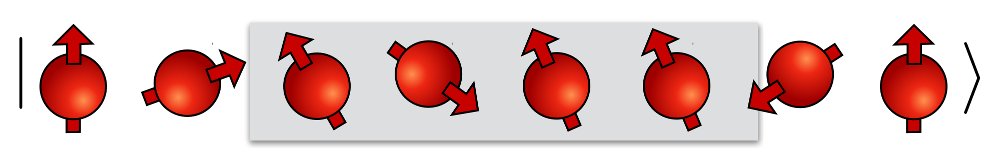
t

$\rho_{\mathrm{Block}} = \rho_{\mathrm{Thermal}}$



$U_t = \exp(-itH)$

[non-integrable]



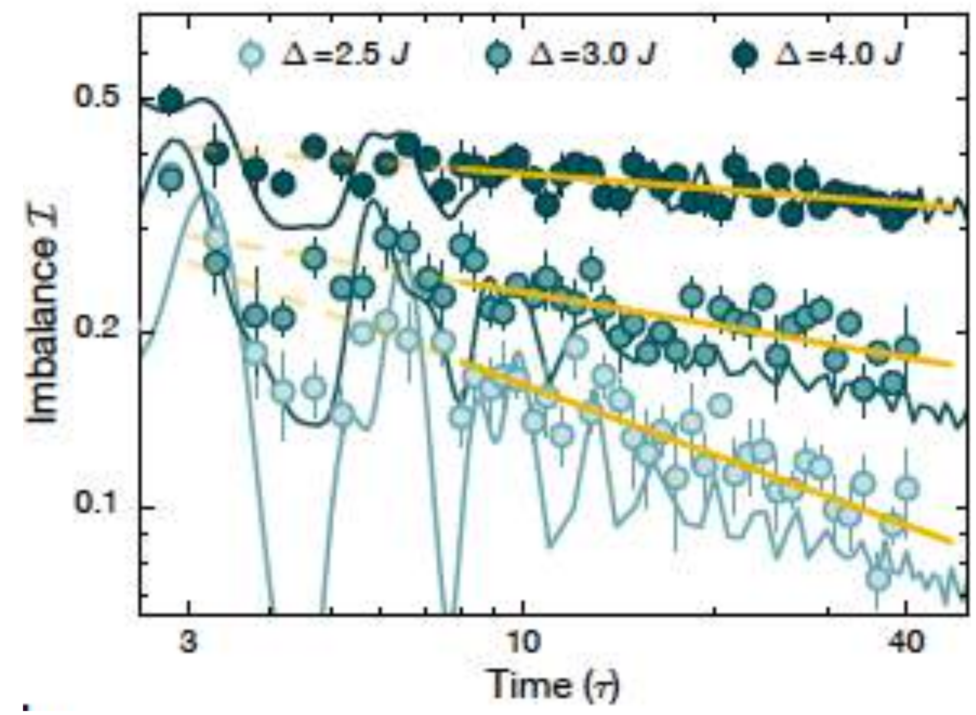
Closed quantum system

[Srednicki, Deutsch, Rigol]

Entanglement accumulated during time evolution

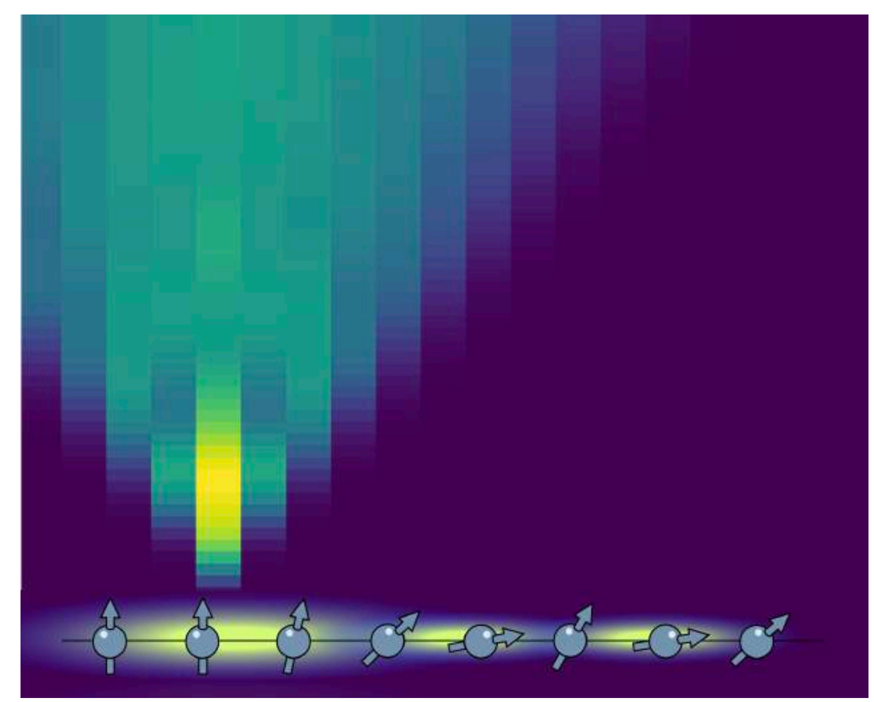
Characterizing thermalization dynamics

Long times: emergent hydrodynamic relaxation $\partial_t e - D\nabla^2 e = \nabla f$



Weakly coupled systems:
Quasi-particles → Boltzmann equation

Strongly coupled systems:
Dynamics of complex quantum-many body problem!



Overview

(1) Entanglement growth following a quantum quench

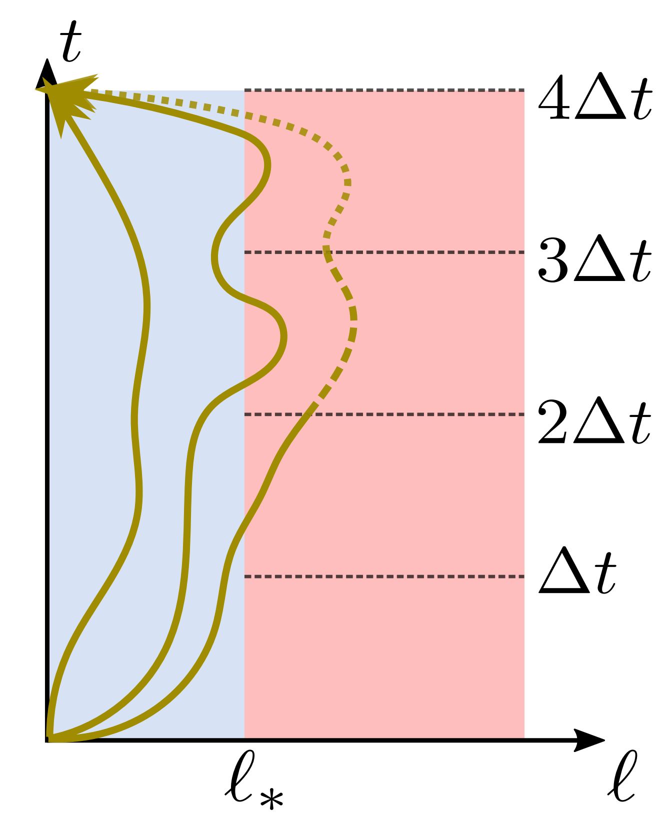
- Diffusive growth of Renyi entropies $S_{\alpha>1}$ in systems with diffusive transport
  [**Rakovszky**, FP, von Keyserlingk, PRL **122**, 250602 (2019)]

$S_1 \propto t$

$S_{\alpha>1} \propto \sqrt{t}$

(2) Dissipation-assisted operator evolution method for capturing hydrodynamic transport

- Efficient calculation of spin and energy diffusion constants
  [**Rakovszky**, von Keyserlingk, FP, arxiv:2004.05177]



Measuring the amount of entanglement

Von Neumann entropy (entanglement entropy)

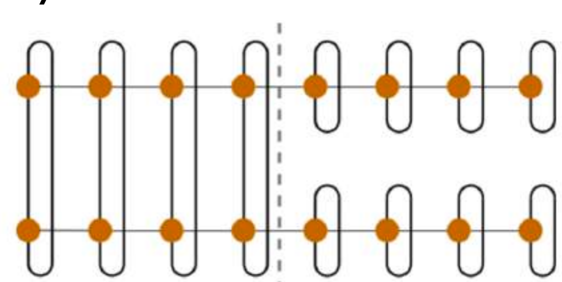
$$S_{vN} = -\,\mathrm{Tr}\rho_{\mathrm{Block}} \log \rho_{\mathrm{Block}}$$

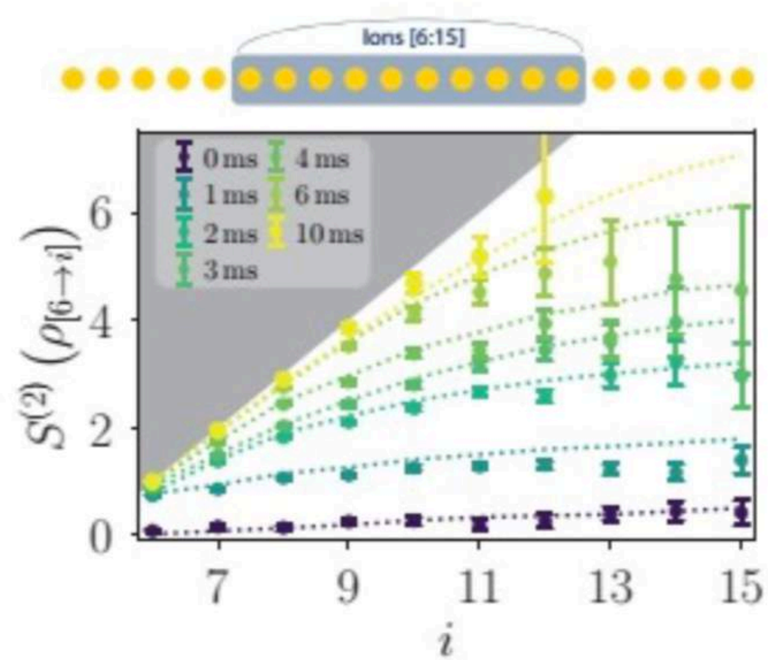
- Convenient for theoretical considerations but not experimentally accessible

Renyi entropies

$$S_\alpha = \frac{1}{1-\alpha} \log \mathrm{Tr}\rho^{\alpha}_{\mathrm{Block}}$$

- Experimentally accessible for $\alpha = 2$
- $S_{vN} = S_1$



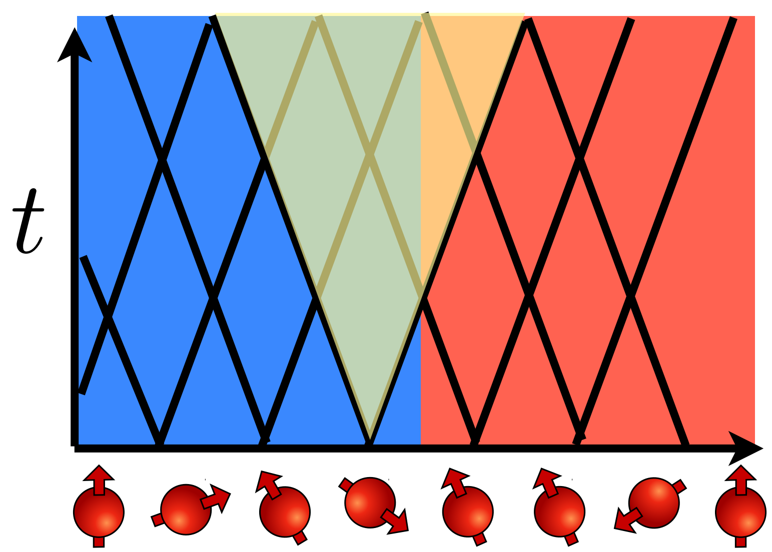


[Brydges et al. arxiv 1806.05747]
[Kaufman et al. Science '16]
[Islam et al. Nature '15]

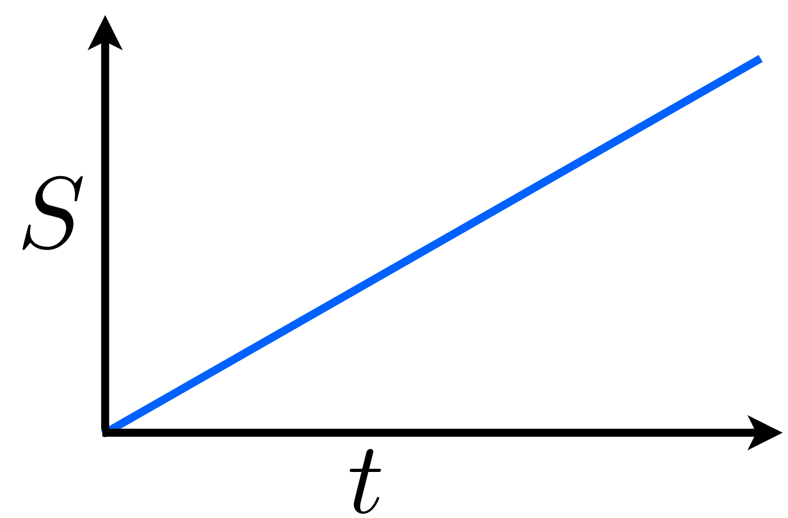
Entanglement growth after a quantum quench

How does the entanglement entropy S_{vN} grow?

- Integrable systems → Quasiparticle picture: linear growth



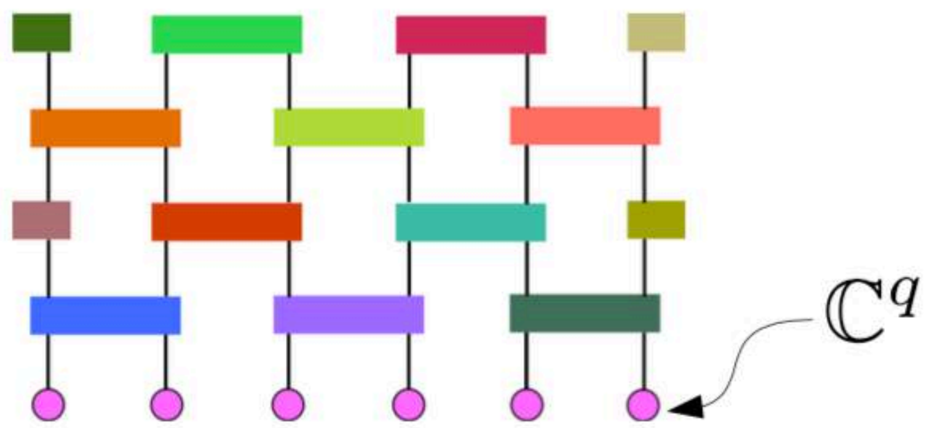
[P. Calabrese and J. Cardy '06]

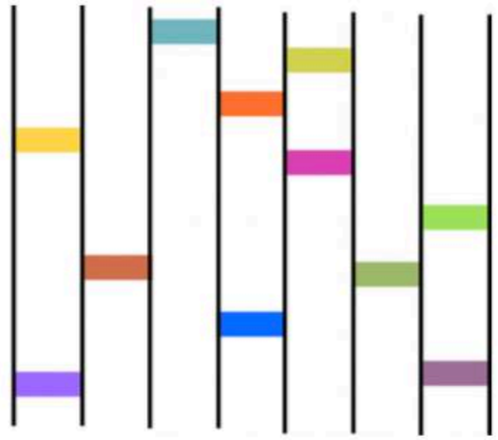


- Linear growth of S_{vN} also holds for systems without quasiparticles [Kim and Huse '13]

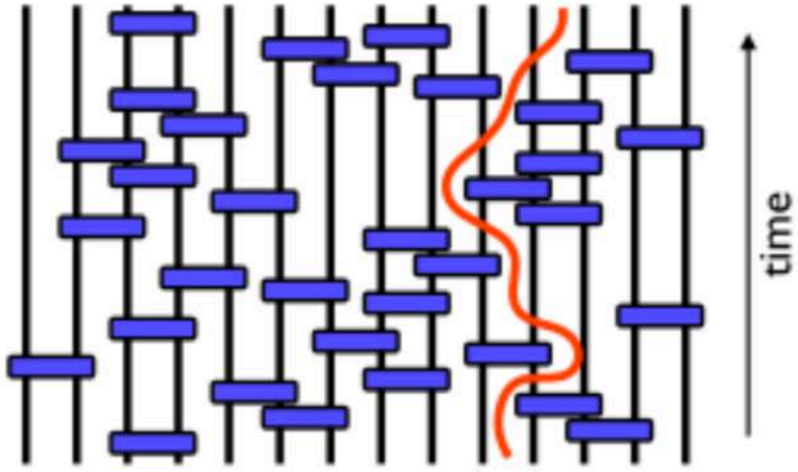
Linear entanglement growth in random circuit models

Each gate is a $q^2 \times q^2$ Haar random unitary





- S_1, S_2 both grow linearly (+ random fluctuations)



Nahum Ruhman, Vijay, Haah: PRX (2017)

Nahum, Vijay, Haah: PRX (2018)

von Keyserlingk, Rakovszky, FP, Sondhi: PRX (2018)

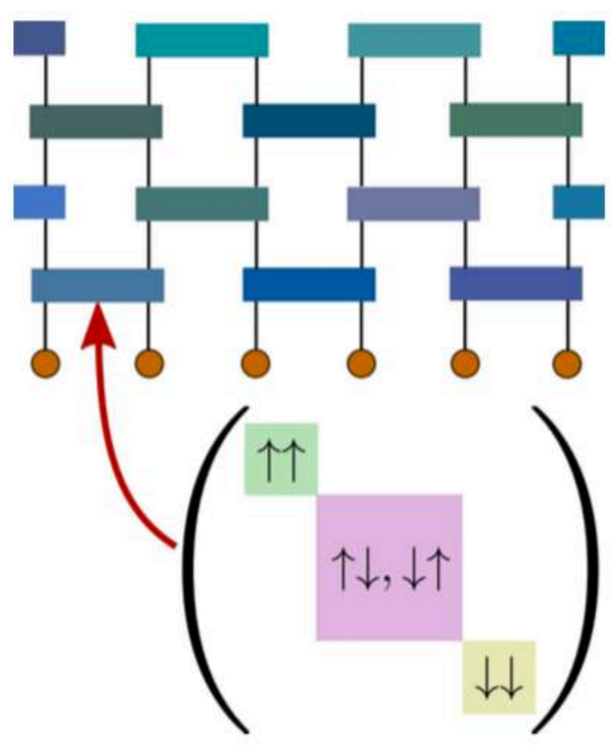
Zhou, Nahum (arXiv 1804.09737)

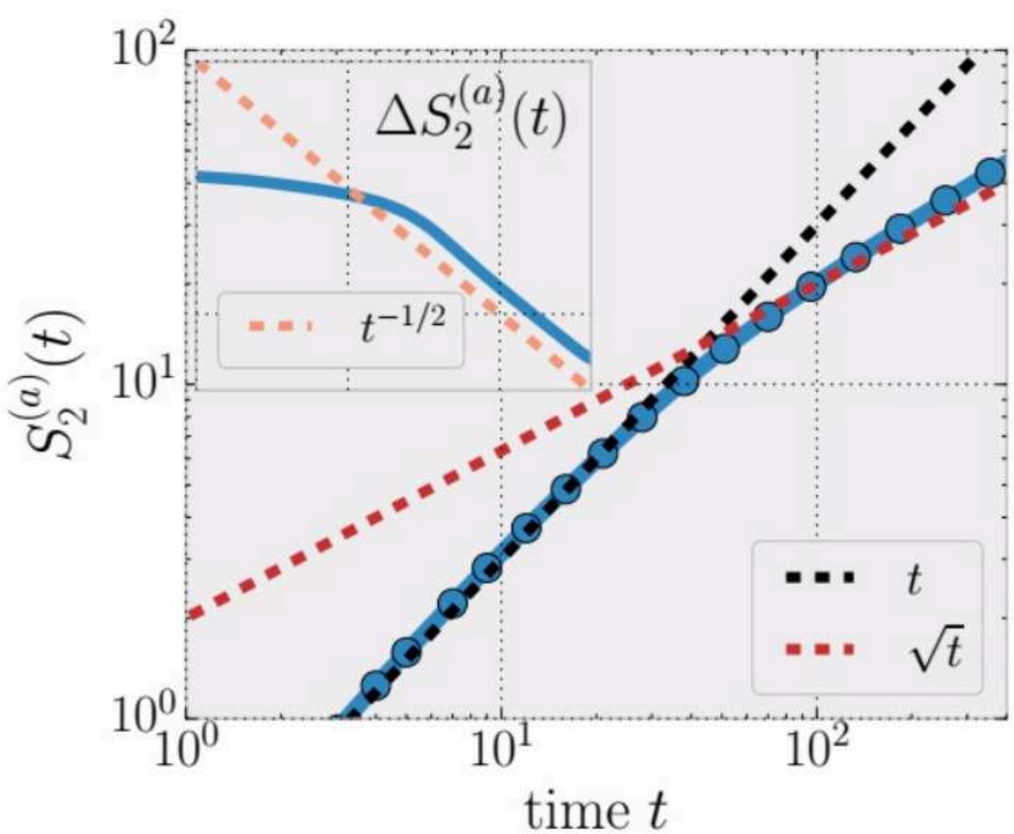
Chan, De Luca, Chalker: PRX (2018)

Growth of Renyi entropies

Conservation laws generically lead to diffusive growth of $S_{\alpha>1}$!

- U(1)-symmetric random circuit
- Maps to "classical" partition function: efficient calculation of the annealed average of 2nd Rényi entropy

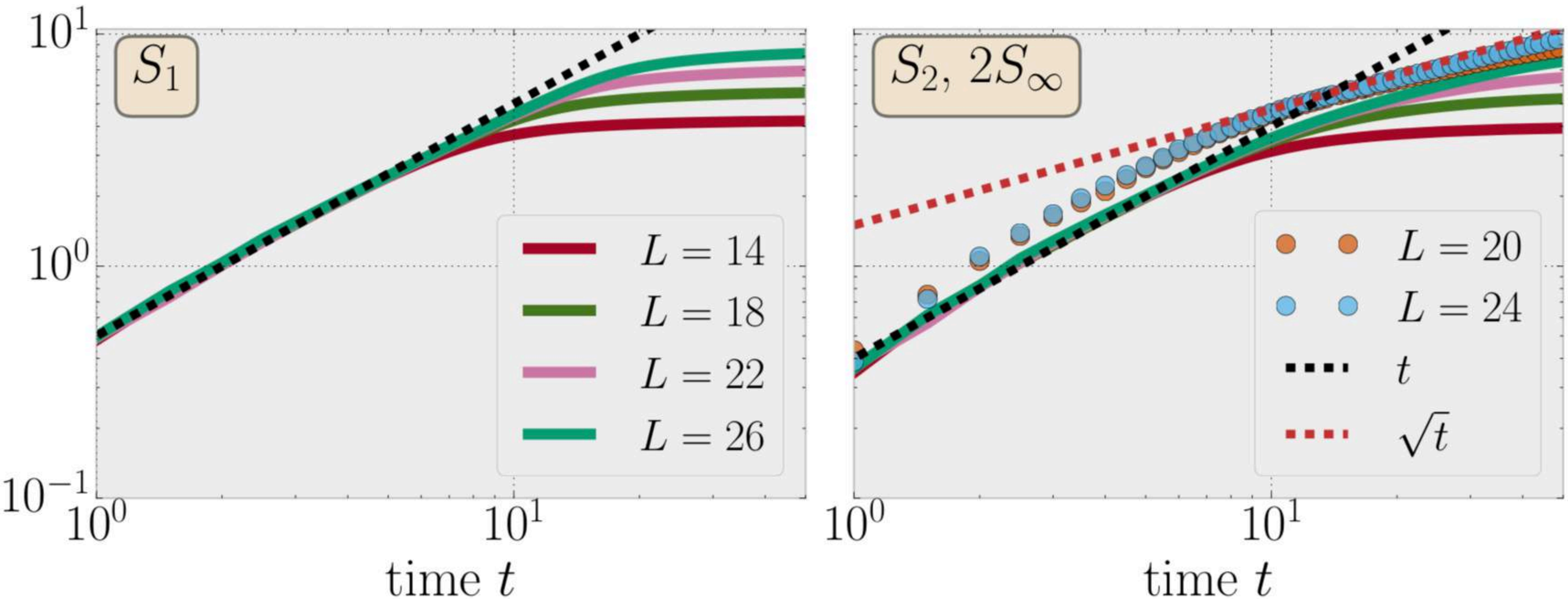




[**Rakovszky**, FP, von Keyserlingk, PRL **122**, 250602 (2019)]

Entanglement growth after a quantum quench

Same behavior in a Hamiltonian with only energy conservation $H = J\sum_r Z_r Z_{r+1} + h_z Z_r + h_x X_r$



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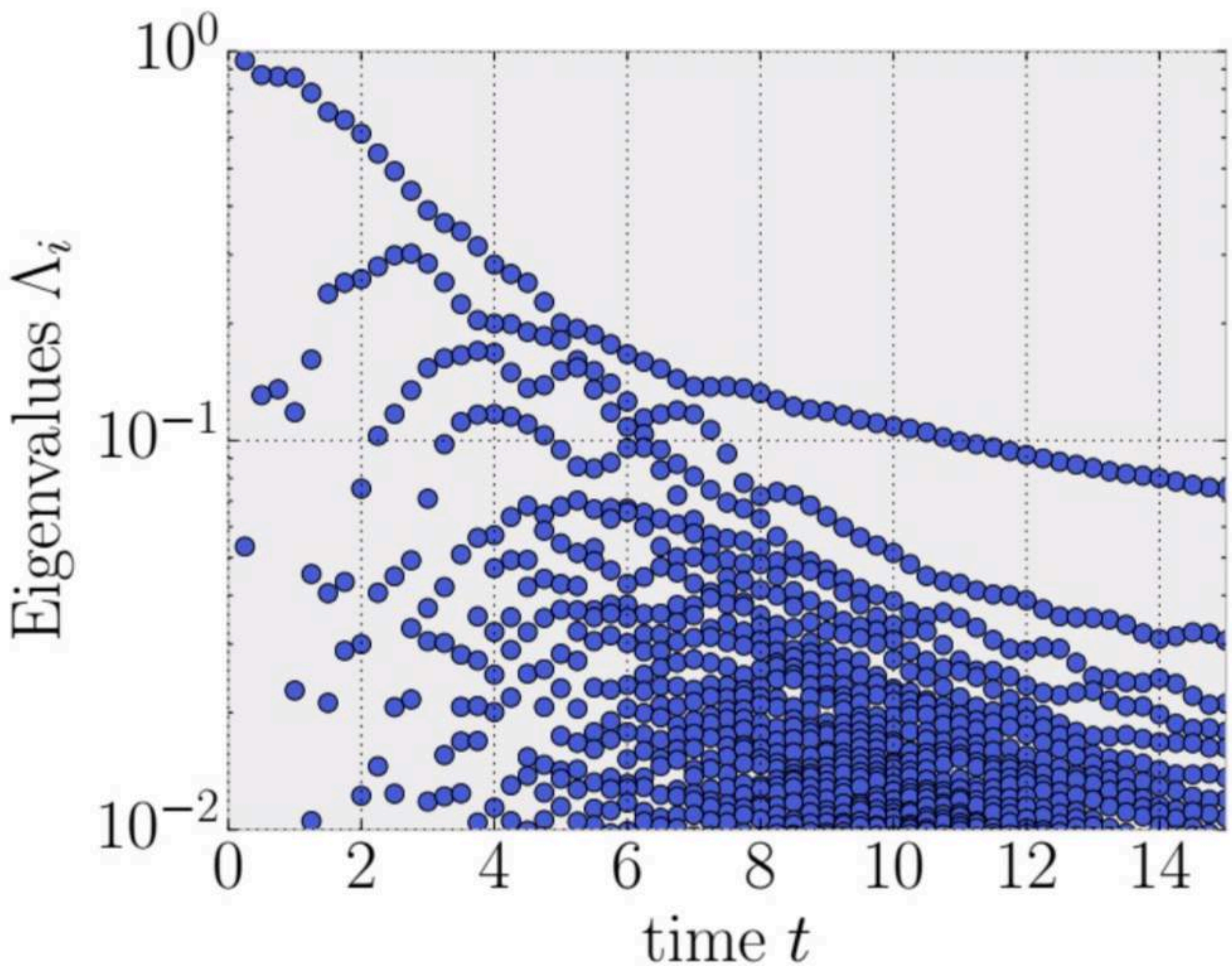
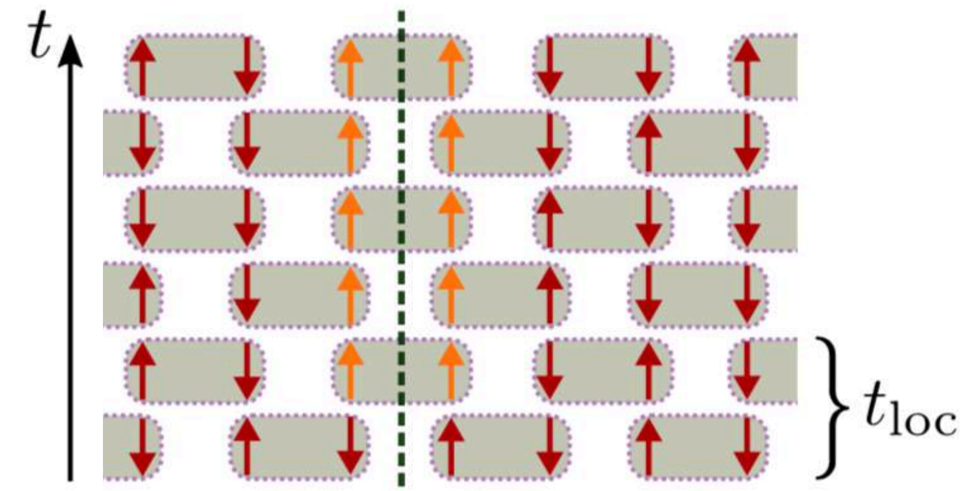


rare states

[**Rakovszky**, FP, von Keyserlingk, PRL **122**, 250602 (2019)]

Intuitive picture

Spin 1/2 chain with S^z conservation



(1) Write state as sum over histories in S^z basis: $|\Psi(t)\rangle = \sum_{\{\sigma(\tau)\}} A(\{\sigma(\tau)\})|\sigma\rangle$

(2) Split sum into two parts $|\Psi(t)\rangle = c_0|\phi_0\rangle + c_1|\phi_1\rangle$ with $|\phi_0\rangle \leftrightarrow \{\sigma(\tau)|\sigma_x = \sigma_{x+1} = \uparrow \text{ if } \tau \geq t_{\text{loc}}\}$. Diffusion: only down spins within distance $\mathcal{O}(\sqrt{t})$ can spoil the rare region $\to |c_0|^2 \propto e^{-\gamma\sqrt{Dt}}$

(3) Eckart-Young theorem: if $|\phi_0\rangle$ has Schmidt rank χ
$\chi\Lambda^2_{\max} \geq |\langle\phi_0|\Psi(t)\rangle|^2 = |c_0 + c_1\langle\phi_0|\phi_1\rangle|^2 \sim e^{-2\gamma\sqrt{Dt}}$

$S_{\alpha>1}$: **Rare events yield diffusive growth**

$S_{\alpha\leq 1}$: **Dominated by the mean, yielding ballistic growth**

Related work:
Huang, arXiv:1902.00977

[**Rakovszky**, FP, von Keyserlingk, PRL **122**, 250602 (2019)]

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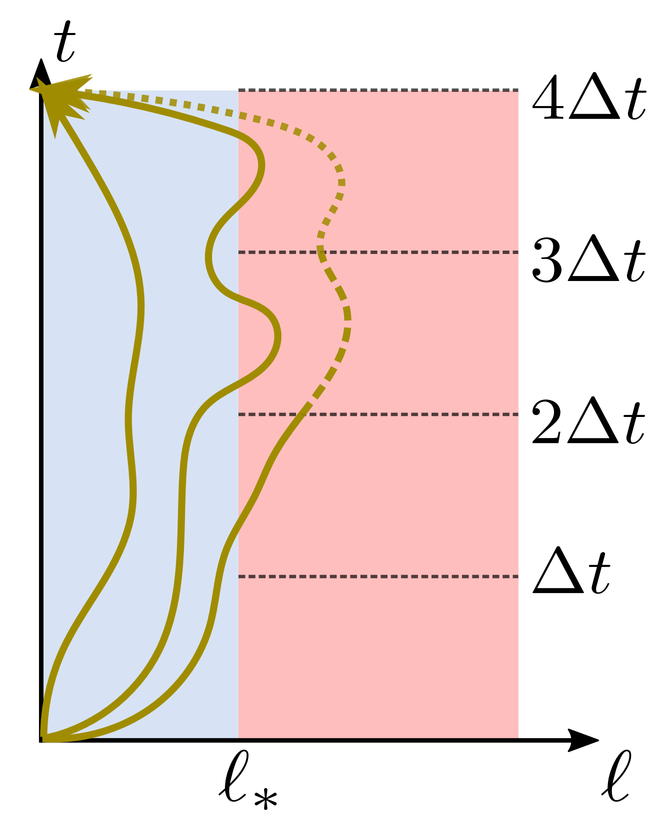
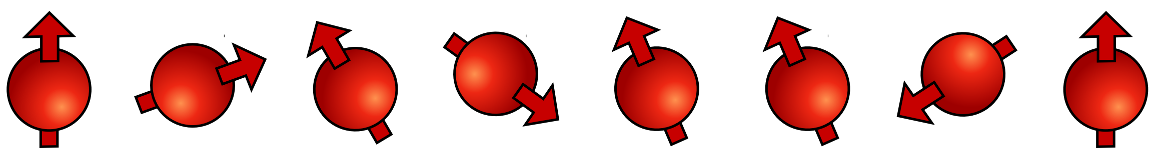


Numerical complexity of many-body dynamics

Directly simulate the time evolution within the full many-body Hilbert space $|\psi(t)\rangle = e^{-itH}|\psi(0)\rangle$



- Complexity $\propto \exp(L)$
- Sparse methods (dynamical typicality) up to ~30 spins

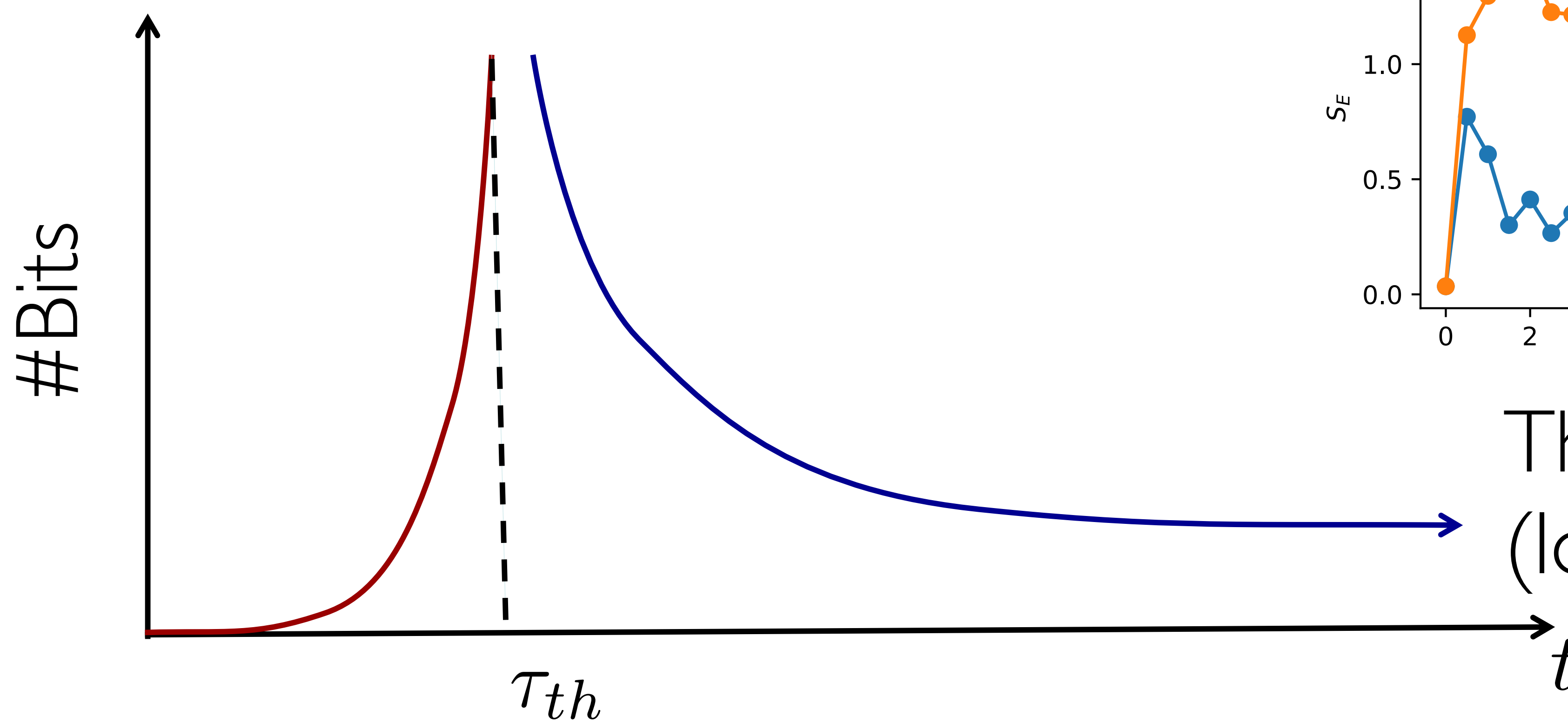
10 spins dim=1'024
20 spins dim=1'048'576
30 spins dim=1'073'741'824
40 spins dim=1'099'511'627'776

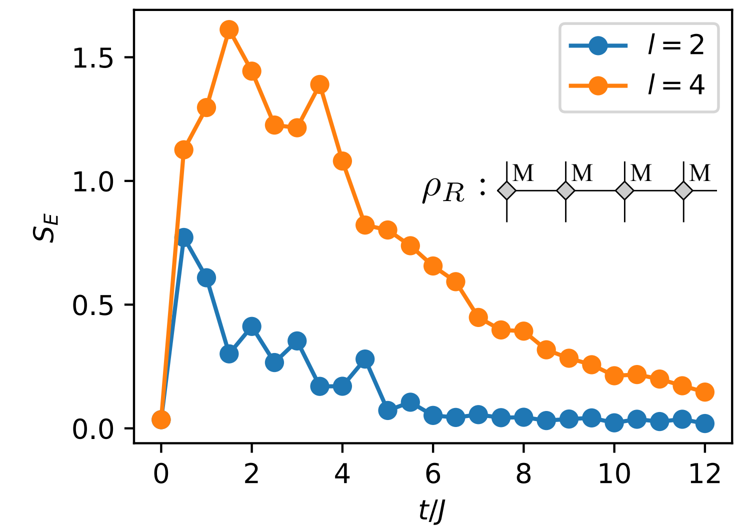
Matrix-Product State based numerics

- Complexity $\propto \exp(t)$

“Information paradox"

Quantum quench from product state





Thermal state (locally)

How to truncate entanglement without sacrificing crucial information on physical (local) observables?

Approaches that still need to demonstrate ability to capture the correct hydro transport:

[White et al.: PRB 2018] [Schmitt, Heyl: SciPost 2018] [Krumnow et al.: arXiv:1904.11999]

[Wurtz et al.: Ann. Phys. 2018] [Parker et al., PRX 2019] [Leviatan et al., arXiv:1702.08894]

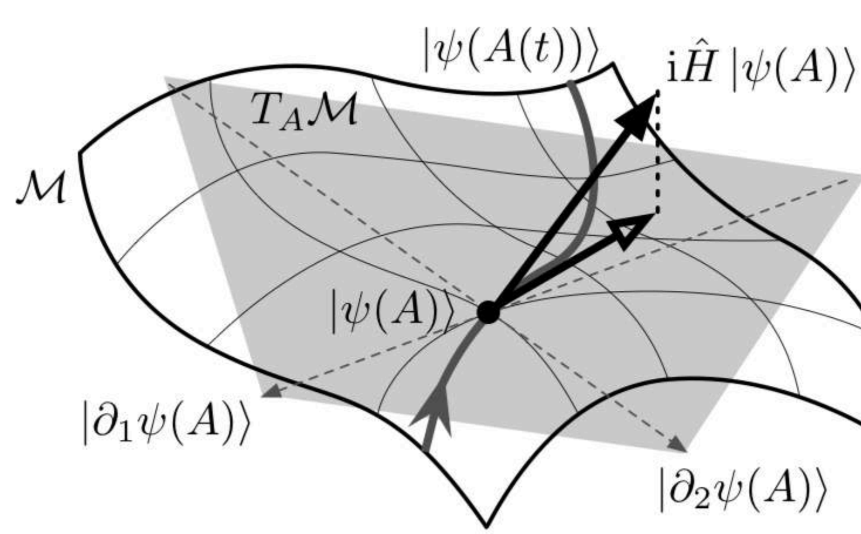
Time-dependent variational principle (TDVP)

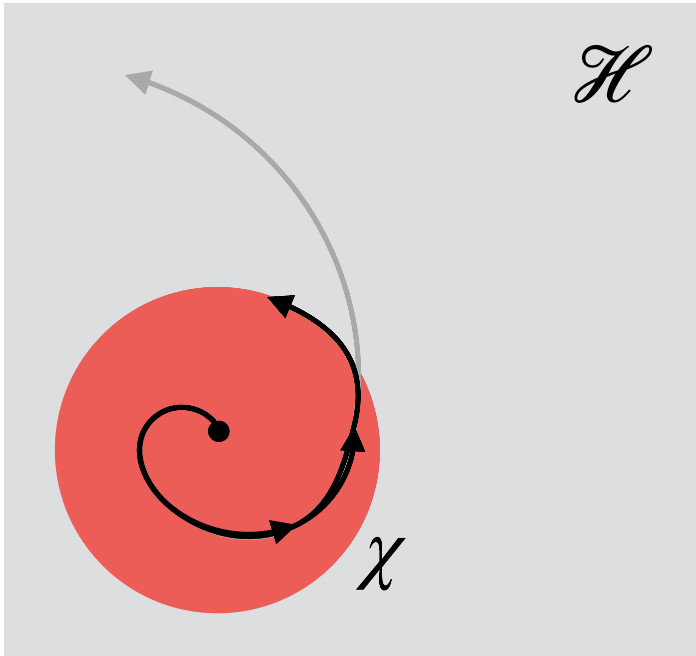
Variational manifold: MPS states with fixed bond dimension

$$\psi_{j_1,j_2,j_3,j_4,j_5} = \quad A^{[1]j_1}_{\alpha} \; A^{[2]j_2}_{\alpha\beta} \; A^{[3]j_3}_{\beta\gamma} \; A^{[4]j_4}_{\gamma\delta} \; A^{[5]j_5}_{\delta}$$

Classical Lagrangian $\mathcal{L}[\alpha, \dot{\alpha}] = \langle \psi[\alpha] \,|\, i\partial_t \,|\, \psi[\alpha] \rangle - \langle \psi[\alpha] \,|\, H \,|\, \psi[\alpha] \rangle$

Efficient evolution using a projected Hamiltonian [Haegeman et al. '11, Dorando et al. '09]





- **Global conservation laws (energy, particles,…)**

[see also: Thermofield purification of the density matrix,
Hallam, Morley, and A. G. Green '19]

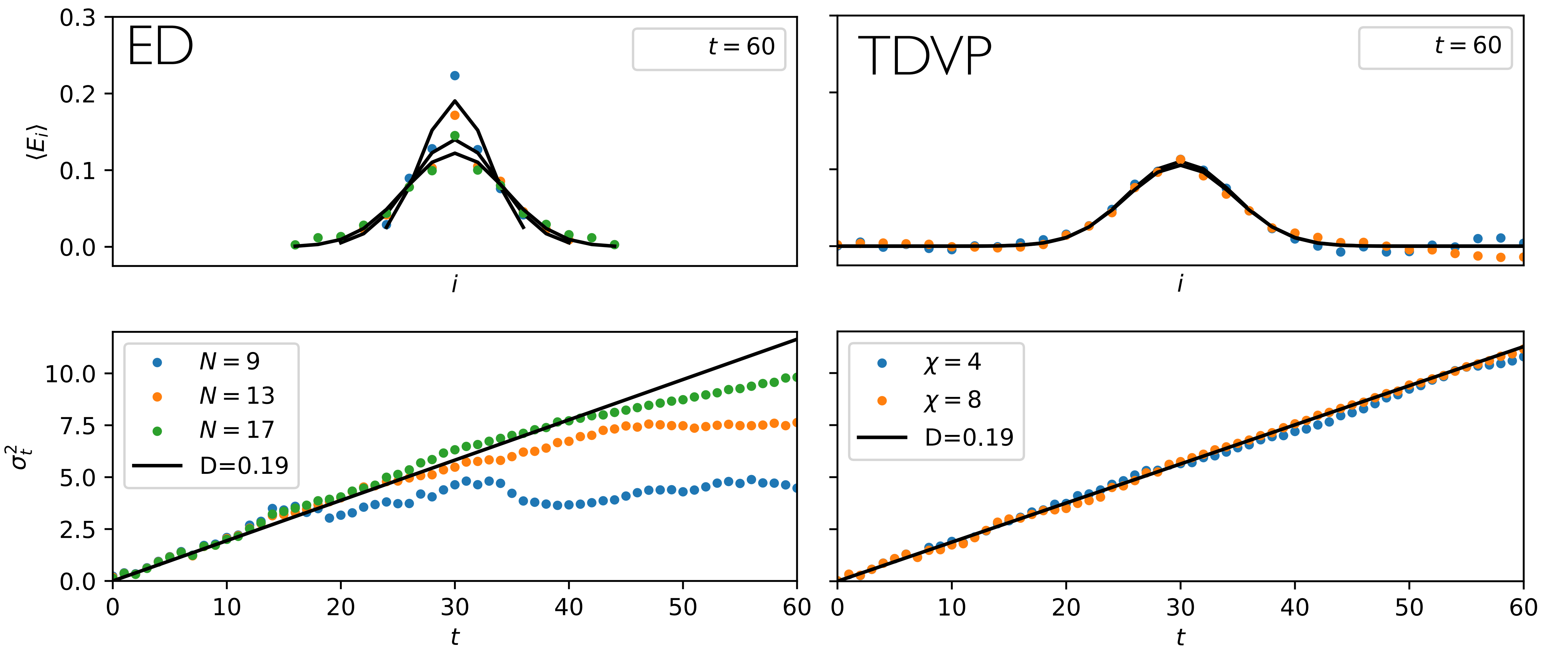
[Leviatan, FP, Bardarson, Huse, Altman, arXiv:1702.08894]

Time-dependent variational principle (TDVP)

Ising model $$H = \sum_i J S_i^z S_{i+1}^z - h_\perp S_i^x - h_{||} S_i^z$$

Ensemble of initial states: $S^+_{L/2}|\psi(0)\rangle$

Energy relaxation



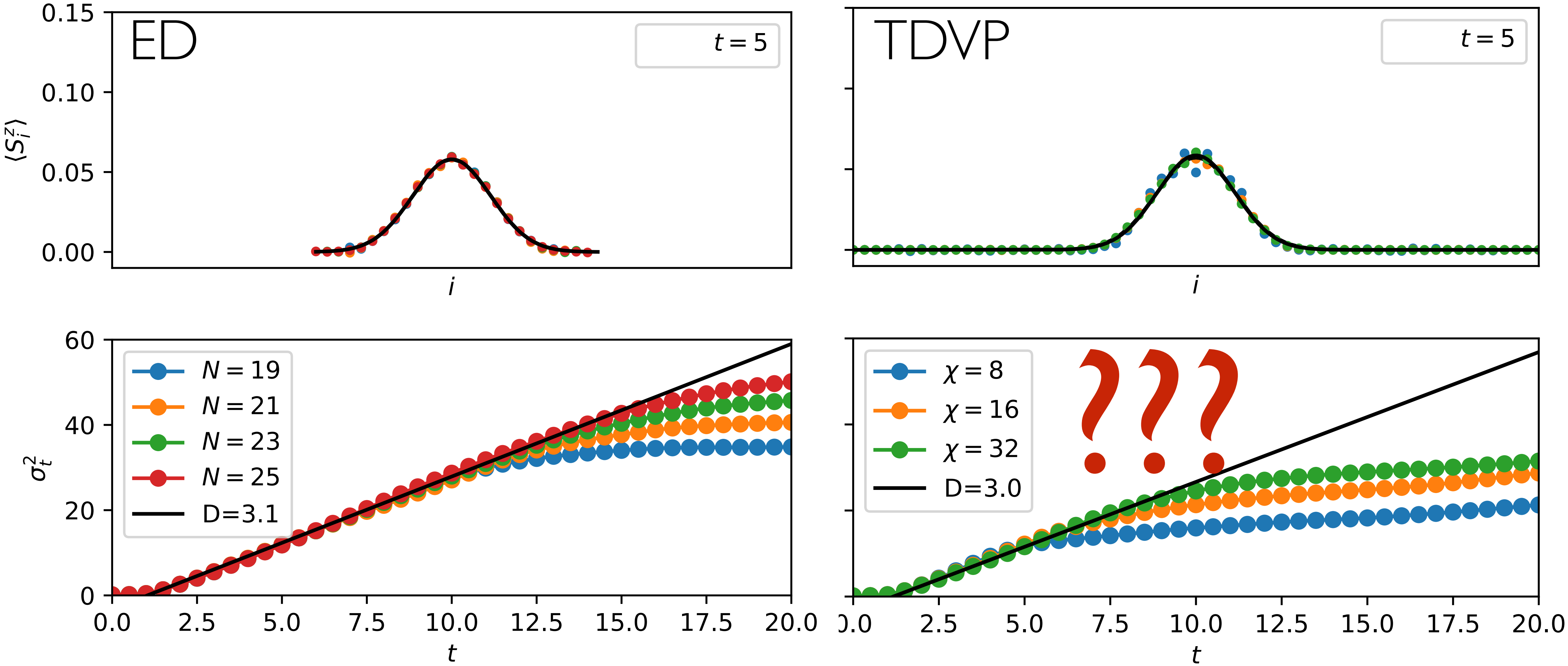
[Leviatan, FP, Bardarson, Huse, Altman, arXiv:1702.08894]

Time-dependent variational principle (TDVP)

XXZ Model $$H = \sum_{i>j} a^{i-j} (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)$$

Ensemble of initial states: $S^+_{L/2}|\psi(0)\rangle$

Sz relaxation

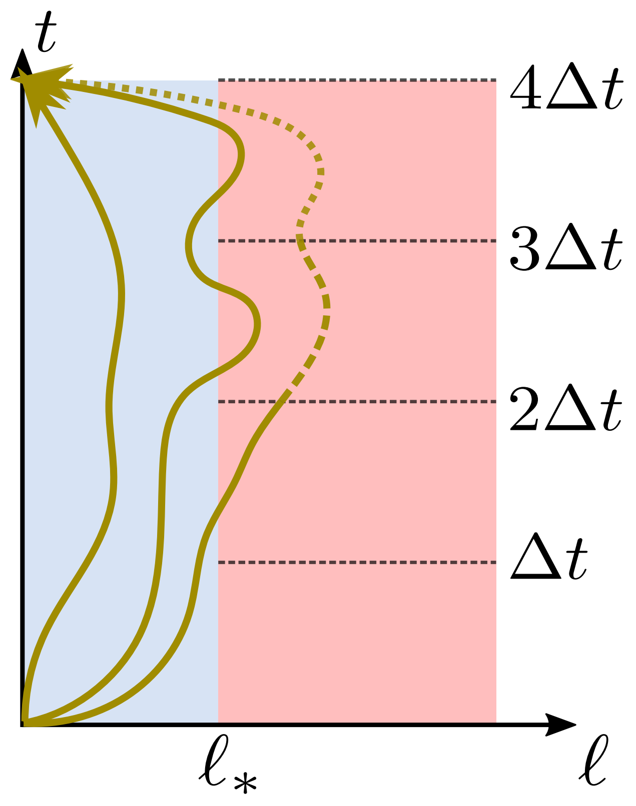


[Leviatan, FP, Bardarson, Huse, Altman, arXiv:1702.08894]

Dissipation-assisted operator evolution method

"Artificial dissipation leads to a decay of operator entanglement, allowing us to capture the dynamics to long times"

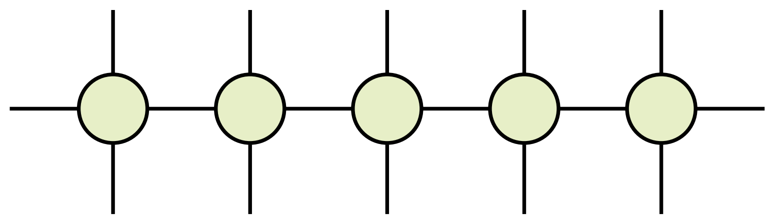
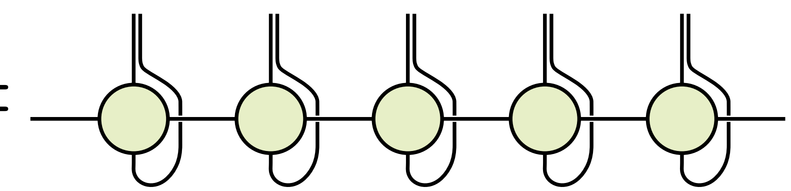
[**Rakovszky**, von Keyserlingk, FP, arxiv:2004.05177]



Obtain dynamical correlations of conserved densities

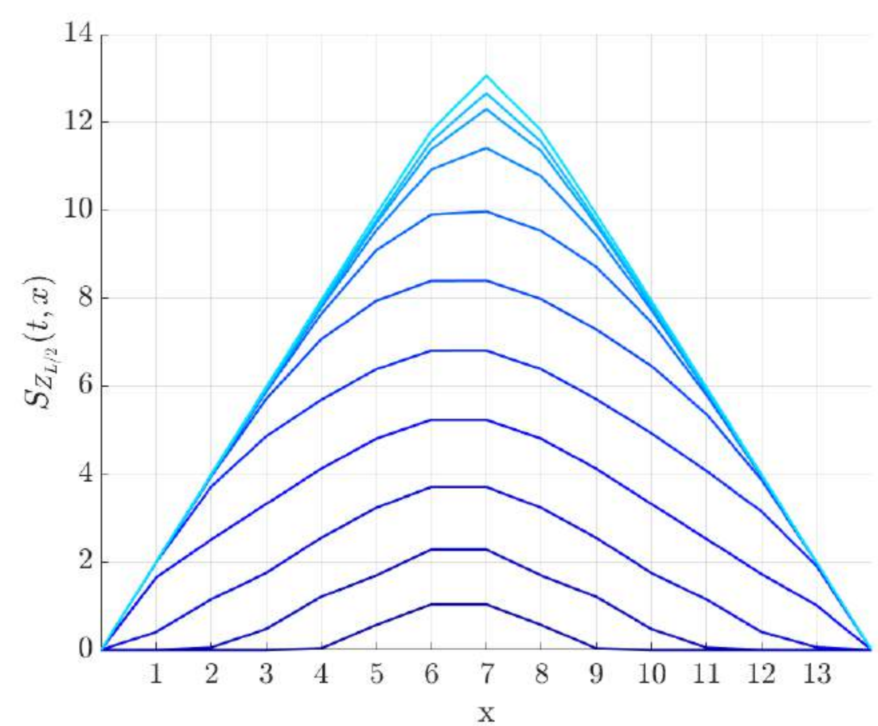
$$C(x,t) \equiv \langle q_x(t) q_0(0) \rangle_{\beta=0} = \langle q_x | e^{i\mathscr{L}t} | q_0 \rangle, \quad \mathscr{L} | q_x \rangle \equiv [H, q_x] = -i\partial_t | q_x \rangle$$

Map operators to states:

$q_j =$  $\rightarrow$ $|q_j\rangle =$ 

Problem: Complexity $\propto \exp(t)$

[Jonay, Huse, Nahum: arXiv:1803.00089]



Artificial dissipation that not affects hydrodynamics

Basis of operators: Pauli strings

$$\mathcal{S} = \ldots ZX\mathbb{1}YX\mathbb{1}\mathbb{1}Y\ldots \quad \longrightarrow \quad |q_0(t)\rangle = \sum_{\mathcal{S}} a_{\mathcal{S}} |\mathcal{S}\rangle$$

Dissipator:

$$\mathcal{D}_{\ell_*,\gamma}|\mathcal{S}\rangle = \begin{cases} |\mathcal{S}\rangle & \text{if } \ell_{\mathcal{S}} \leq \ell_* \\ e^{-\gamma(\ell_{\mathcal{S}}-\ell_*)}|\mathcal{S}\rangle & \text{otherwise} \end{cases}$$

Cutoff length $\ell_* =$ #non-trivial Paulis
(should be larger than support of conserved densities)

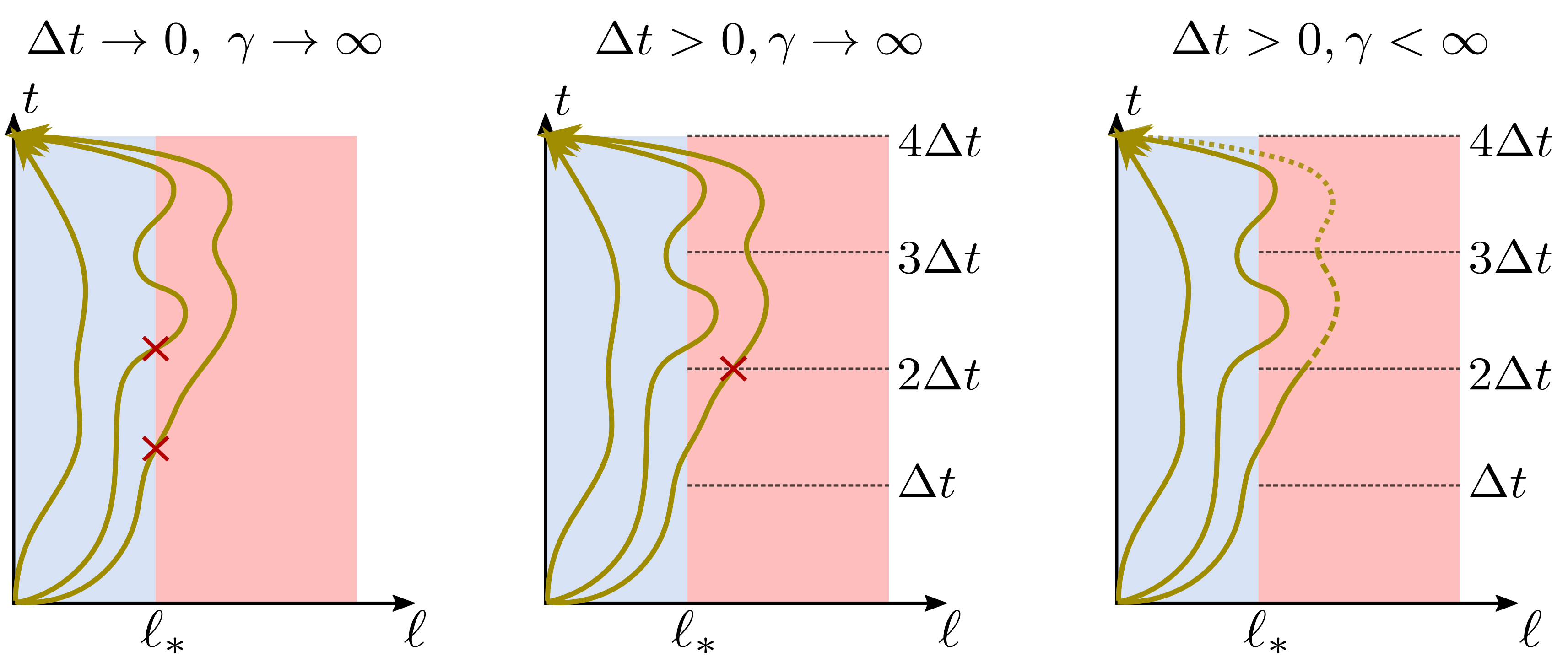
Dissipation strength: γ

[**Rakovszky**, von Keyserlingk, FP, arxiv:2004.05177]

Artificial dissipation that not affects hydrodynamics

Modified evolution: dissipate after every Δt

$$|\tilde{q}_x(N\Delta t)\rangle \equiv \left(\mathscr{D}_{\ell_*,\gamma} e^{i\mathscr{L}\Delta t}\right)^N |q_x\rangle \qquad \mathscr{D}_{\ell_*,\gamma}|\mathcal{S}\rangle = \begin{cases} |\mathcal{S}\rangle & \text{if } \ell_{\mathcal{S}} \leq \ell_* \\ e^{-\gamma(\ell_{\mathcal{S}}-\ell_*)}|\mathcal{S}\rangle & \text{otherwise} \end{cases}$$

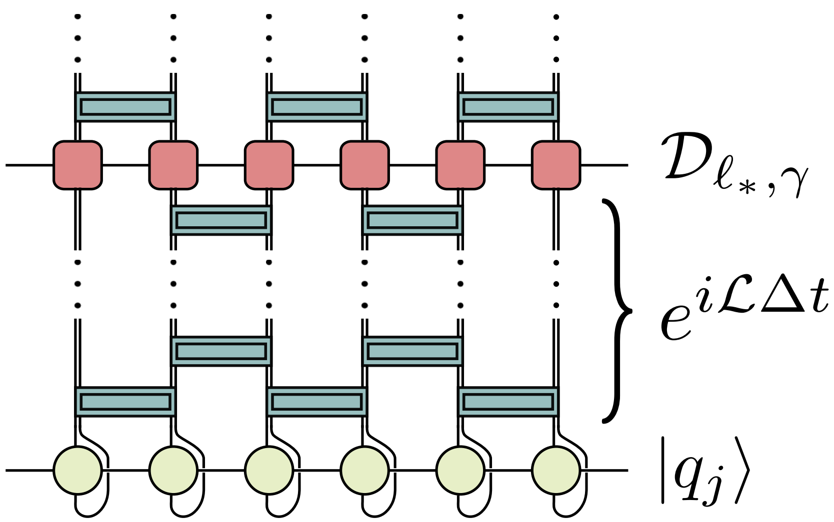


- Key assumption: **backflow** from long to short operators is weak
- Compare: **Short memory time** in Zwanzig-Mori memory matrix

[**Rakovszky**, von Keyserlingk, FP, arxiv:2004.05177]

Dissipation stops growth of operator entanglement

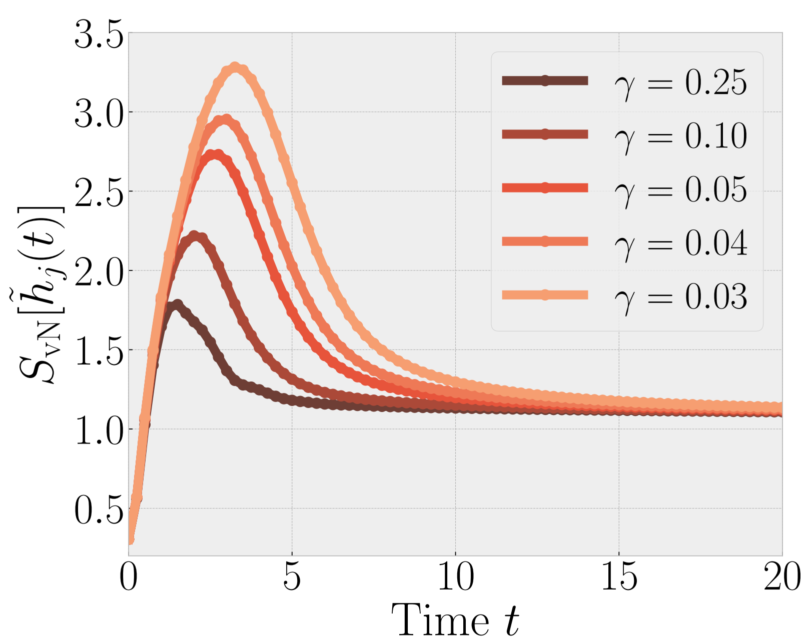
Represent dissipative evolution as tensor network

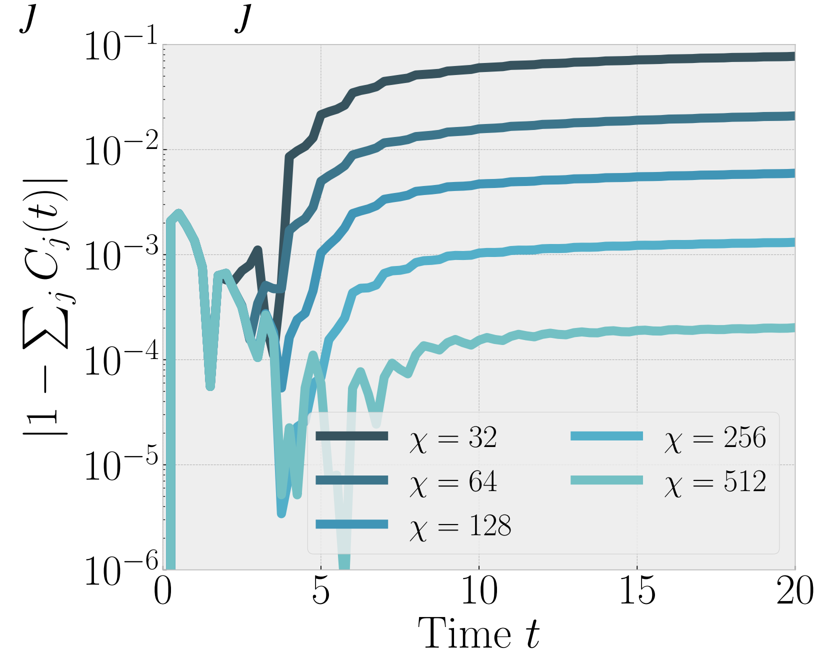


Low-dimensional Matrix-Product Operator

Time Evolving Block Decimation (TEBD) [Vidal '03]

Test on Ising chain: $H = \sum_j h_j \equiv \sum_j g_x X_j + g_z Z_j + (Z_{j-1}Z_j + Z_j Z_{j+1})/2$



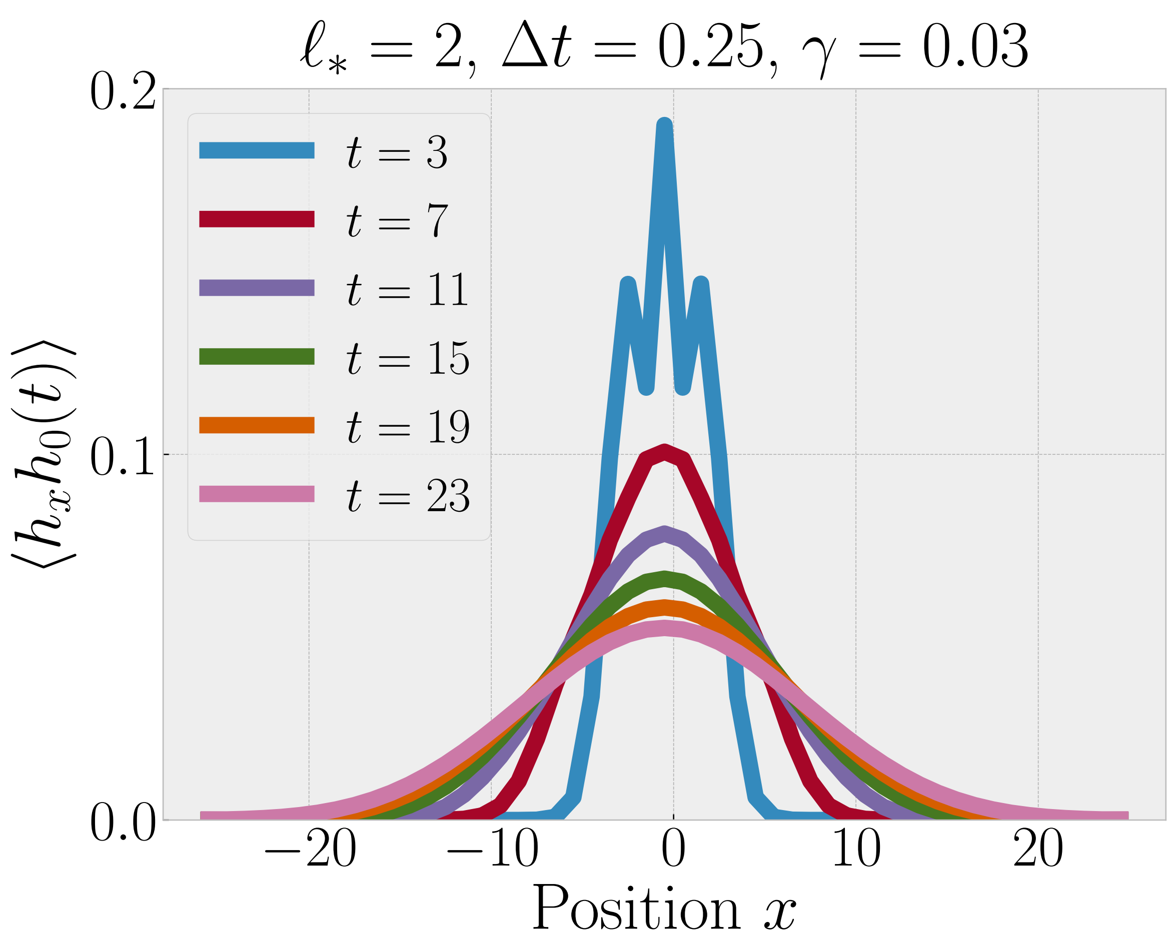


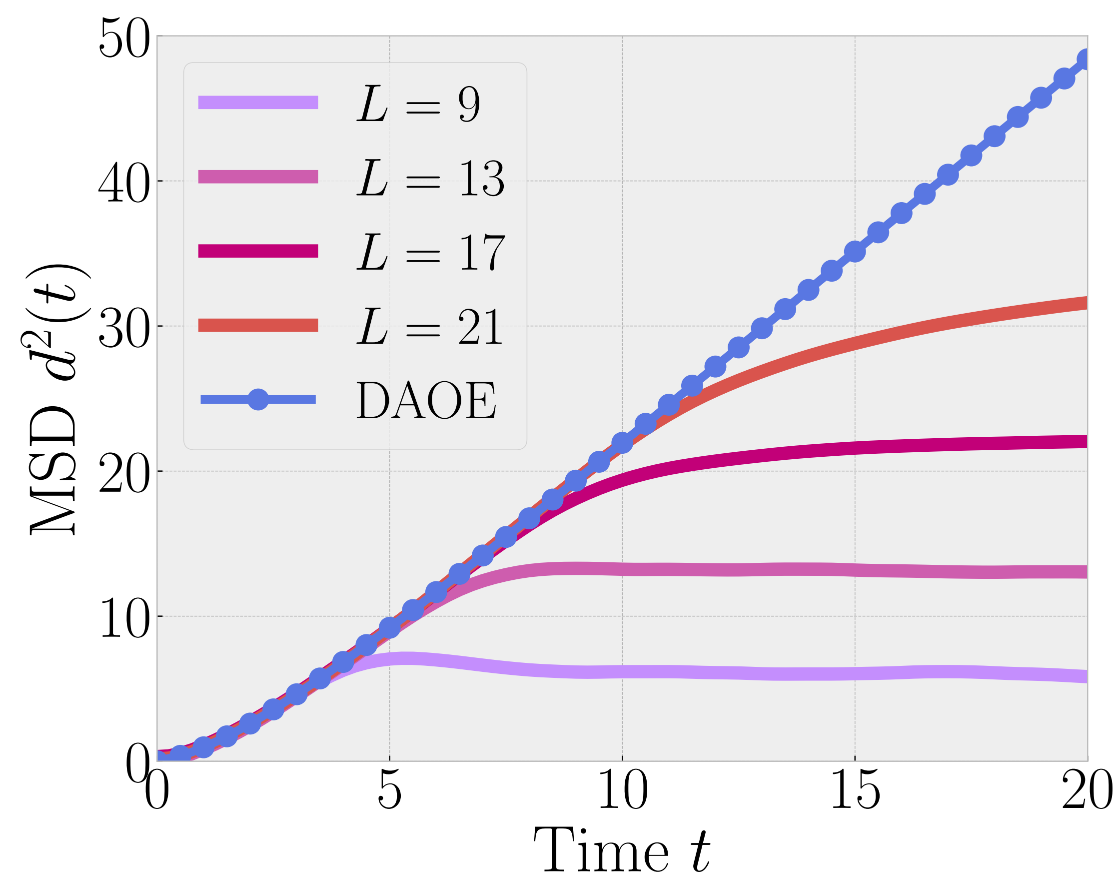
$g_x = 1.4;\ g_z = 0.9045$

[**Rakovszky**, von Keyserlingk, FP, arxiv:2004.05177]

Diffusion constant from mean-square displacement

$$C(x,t) \equiv \langle q_x | \tilde{q}_0(t) \rangle \quad \longrightarrow \quad d^2(t) \equiv \sum_x C(x,t) x^2 \quad \text{(MSD)}$$





Time-dependent diffusion constant: $2D(t) \equiv \dfrac{\partial d^2(t)}{\partial t}$

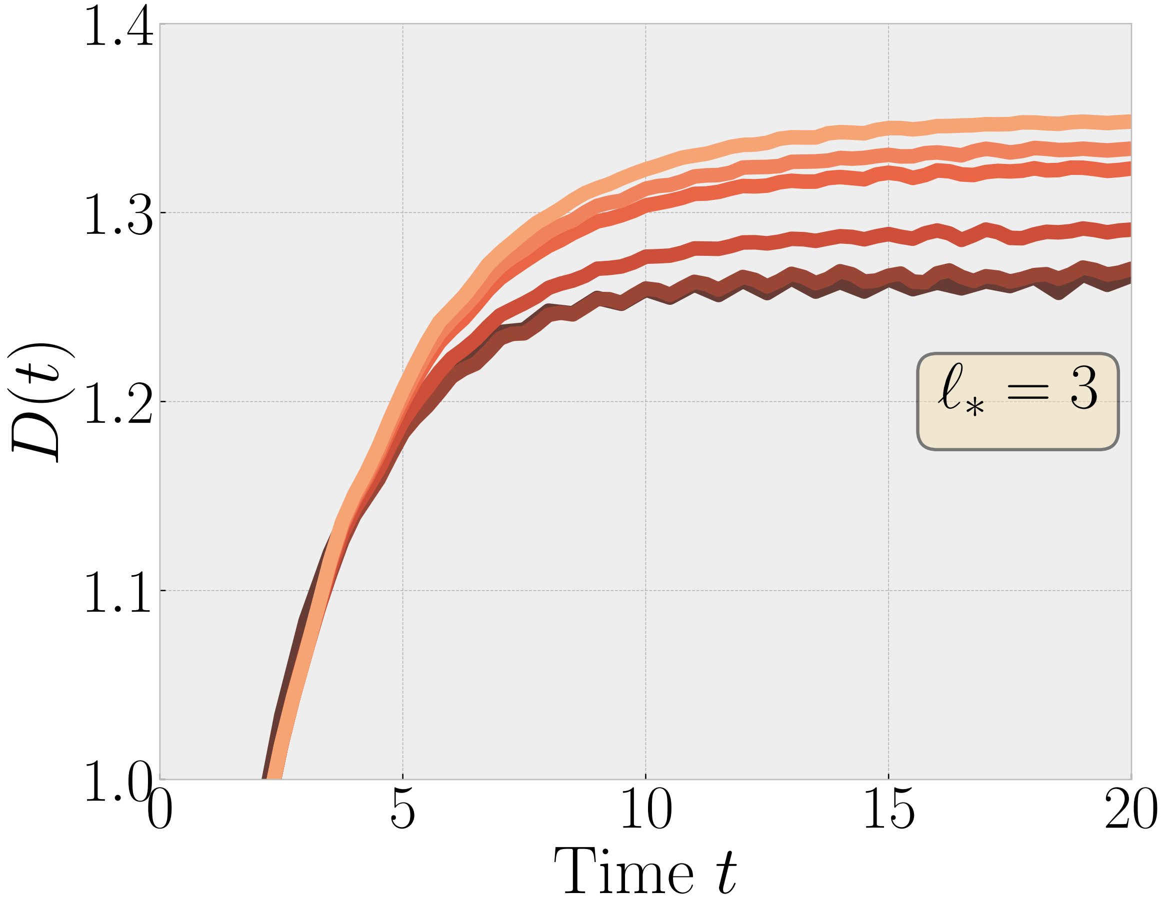
Diffusive transport: $D \equiv \lim_{t \to \infty} D(t)$

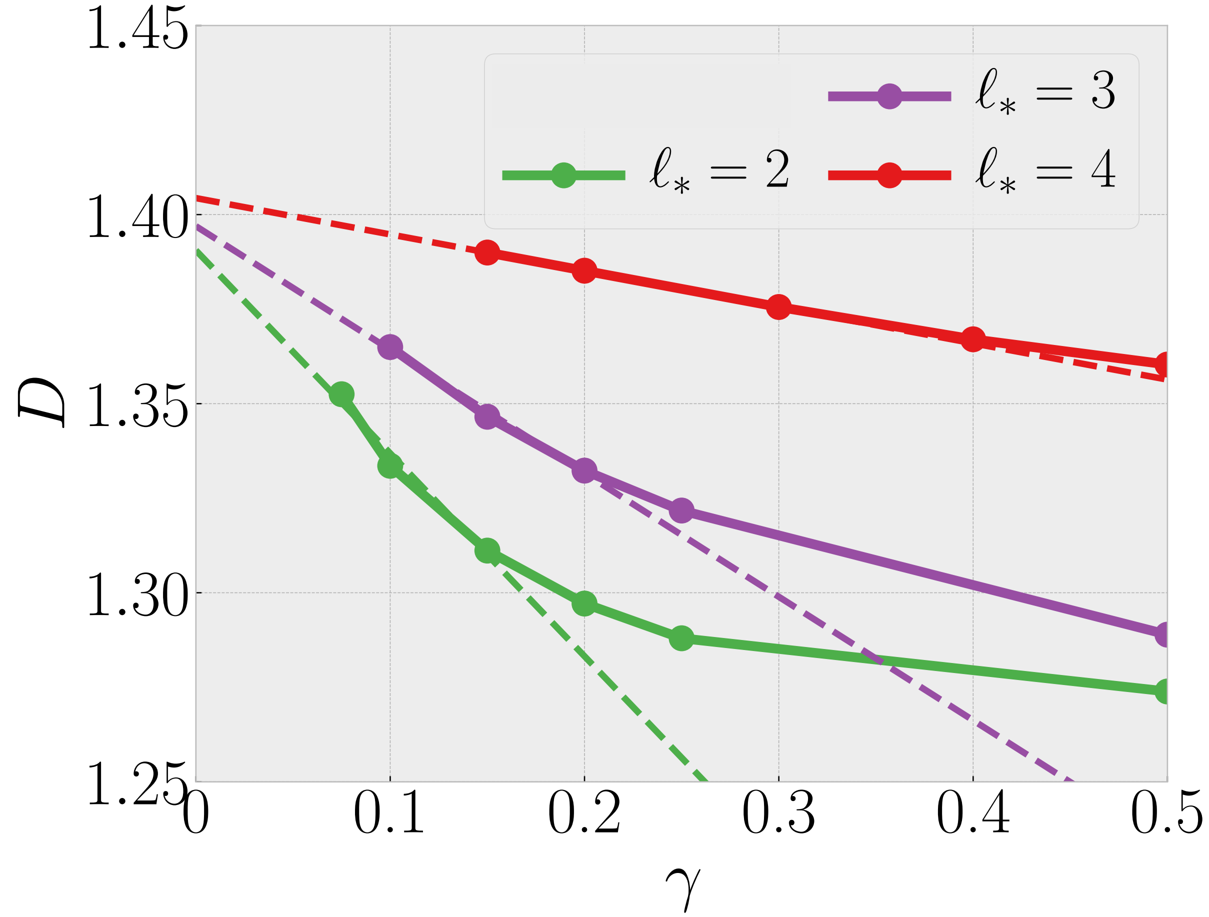
[**Rakovszky**, von Keyserlingk, FP, arxiv:2004.05177]

High precision in various models

Ising: $H = \sum_j h_j \equiv \sum_j g_x X_j + g_z Z_j + \frac{1}{2}(Z_{j-1}Z_j + Z_j Z_{j+1})$

$q_x \equiv h_j$



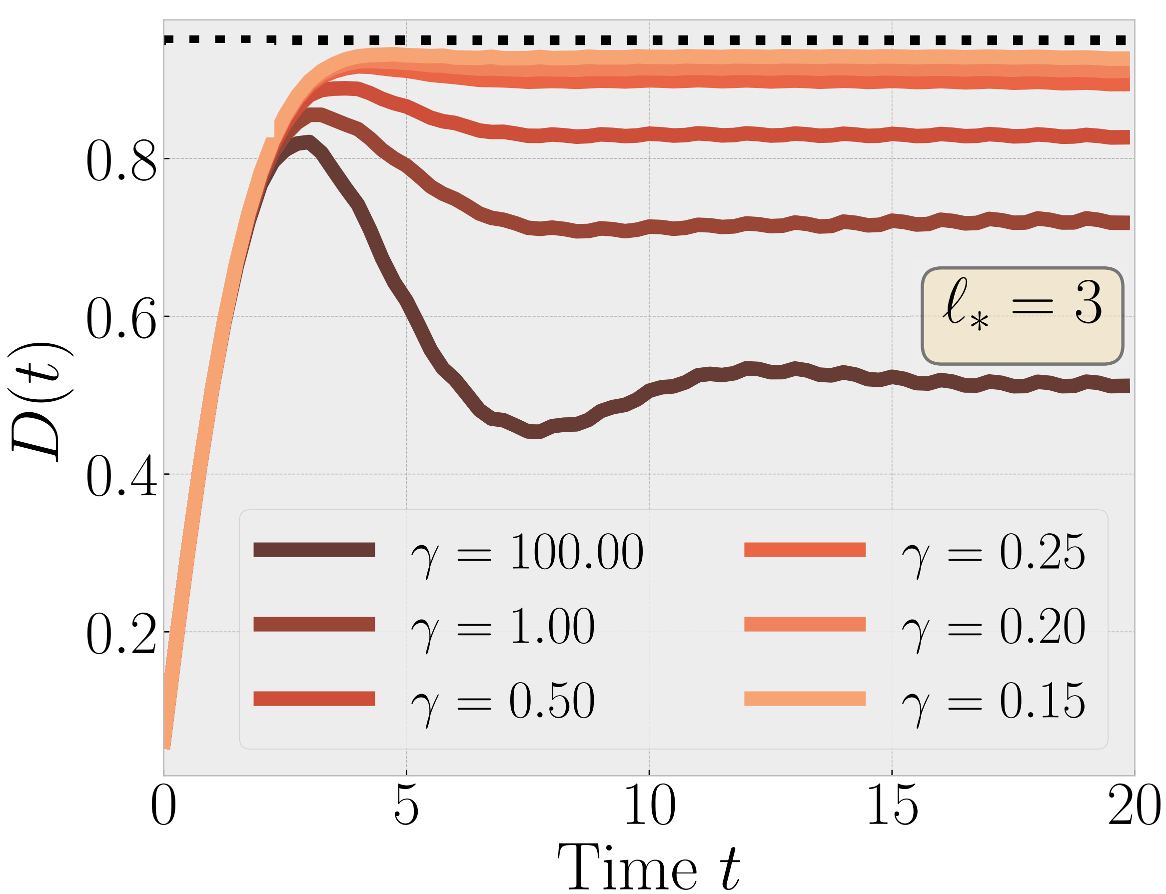


[**Rakovszky**, von Keyserlingk, FP, arxiv:2004.05177]

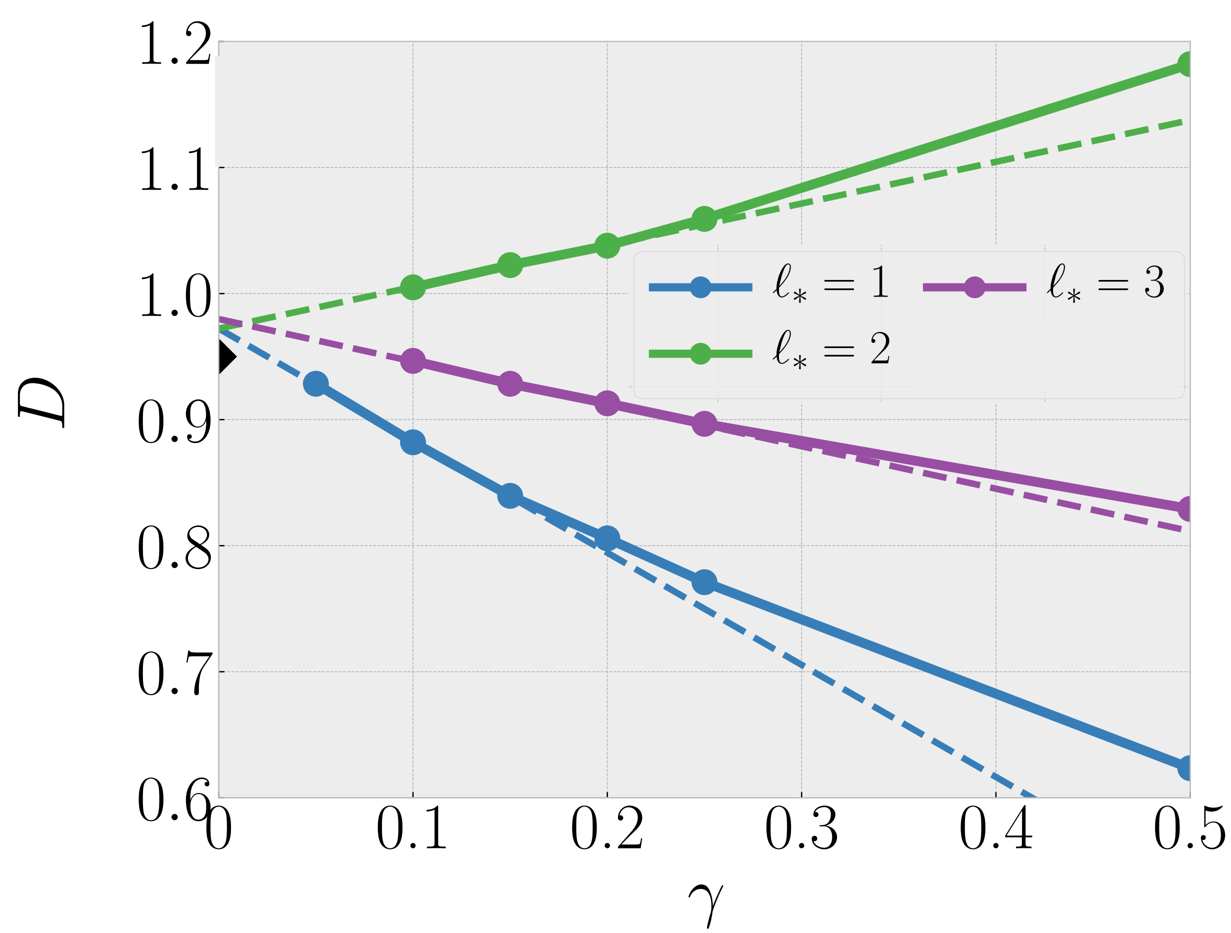
High precision in various models

XX ladder: $$H = \sum_{j=1}^{L} \sum_{a=1,2} \left(X_{j,a} X_{j+1,a} + Y_{j,a} Y_{j+1,a} \right) + \sum_{j=1}^{L} \left(X_{j,1} X_{j,2} + Y_{j,1} Y_{j,2} \right)$$

$q_x \equiv (Z_{j,1} + Z_{j,2})/2$



$D \approx 0.95$ [Steinigeweg et al., '14]



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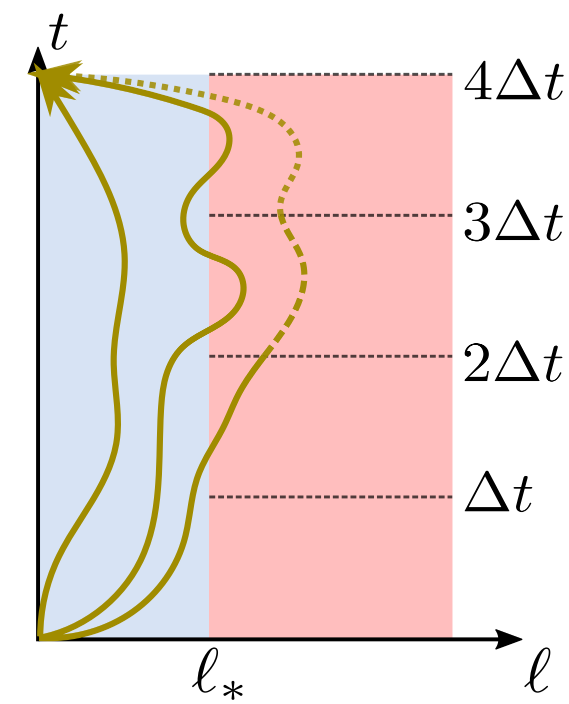

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Thank You!