A brief introduction to quasiparticles in frustrated magnets

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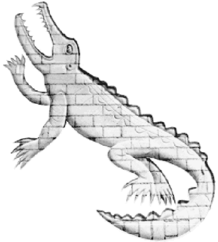
HELMHOLTZ ASSOCIATION
Virtual Institute: New States of Matter and their Excitations

Claudio Castelnovo

TCM group

Cavendish Laboratory

University of Cambridge



17-06-2020

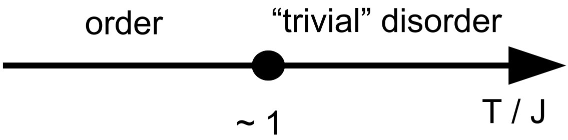
CMP in All the Cities

Outline

- magnetism, frustration and spin liquid behaviour
- modelling spin liquids: general overview
- quasiparticle excitations: 6-vertex and 8-vertex model
  - classical behaviour: deconfinement, fractionalisation, dynamical constraints and entropic interactions
  - quantum behaviour: fractional statistics and dual quasiparticles $\rightarrow$ toric code and quantum spin ice
- quantum spin liquids at finite temperature

  (a prelude to the second talk)
- conclusions

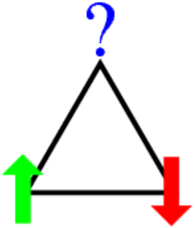
Conventional Magnetism vs Spin Liquids



“trivial”: $T \gtrsim J \Rightarrow$ high-T expansion holds ($\langle S_i S_j \rangle \sim -H_{ij}/T$)

frustration: inability to minimise locally all energy terms $\Rightarrow T_c \ll J$

$H = J \sum_{ij} S_i S_j$ (triang. Ising AFM)



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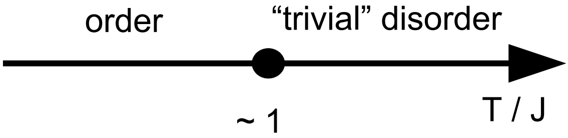

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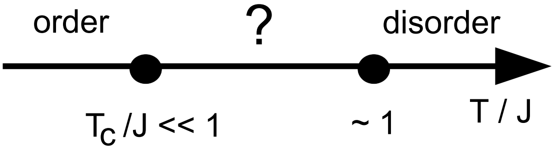
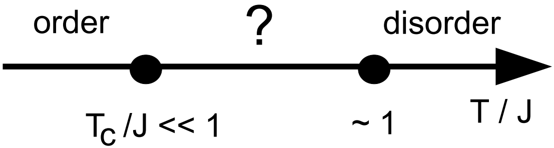


Conventional Magnetism vs Spin Liquids

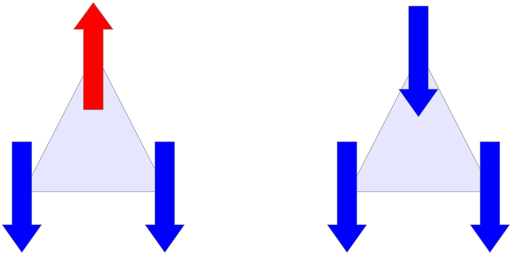


- ~ 1 is typically a crossover (Schottky anomaly)
- no long range order
- non-trivial spin correlations ($T < J \quad \Rightarrow \quad$ ~~$\langle S_i S_j \rangle \sim -H_{ij}/T$~~)

$\longrightarrow$ **spin liquid**

Modelling (classical) spin liquids

example: nn Ising AFM on triangular lattice



2:1 triangles vs 3:0 triangles

energy difference: $\Delta \sim J$

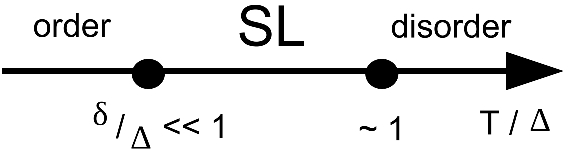
$\Rightarrow$ projects onto mostly 2:1 configurations for $T \lesssim \Delta$

Modelling (classical) spin liquids

generally: $H \sim H_\Delta + H_\delta$

- leading contribution (H_Δ) projects onto subset of configuration space (no spontaneous symmetry breaking) for $T \lesssim \Delta$
- possible subleading contributions (H_δ) cause ordering for $T \lesssim \delta \ll \Delta$

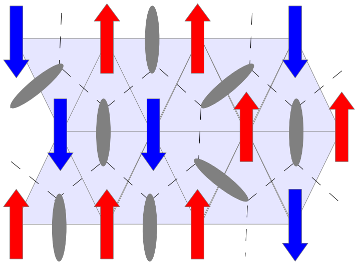
(triang. nn Ising AFM: $H_\delta = 0$)



Effective dimer description

- for $T \lesssim \Delta$, mostly 2:1 triangles

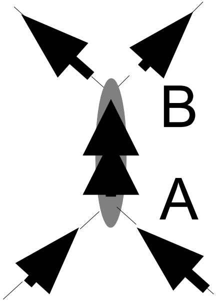
- ferro bonds equivalent to dimers on dual honeycomb lattice



leading to:

- extensive degeneracy
- non-trivial correlations

Emergent gauge symmetry and dipolar correlations



- dimer = flux 2 from A to B
- no-dimer = flux 1 from B to A
- dimer constraint = divergenceless condition

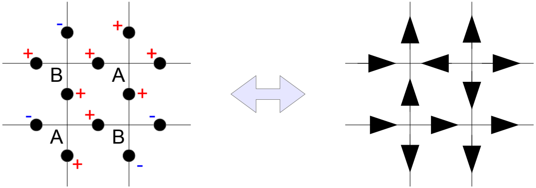
$\longrightarrow$ emergent gauge field

Henley AR 2010

$\Rightarrow$ 2D dipolar correlations: $\langle \text{flux flux} \rangle \sim \langle \text{dimer dimer} \rangle \sim \langle S\, S \rangle$

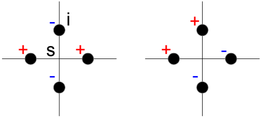
Elementary excitations of H_Δ

for convenience:
Ising model on bonds of square lattice

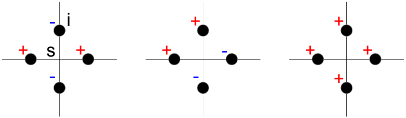


consider Hamiltonians that result in leading projection term H_Δ favouring:

- $\sum_{i \in s} \sigma_i = 0$ (6 vertex model)



- $\prod_{i \in s} \sigma_i = 1$ (8 vertex model)



Six-vertex vs eight-vertex model

- extensively degenerate

Pauling's entropy estimate:
$2N$ spins, N sites
n out of 16 ($n = 6, 8$) minimal energy configurations per site

$$S \sim \ln \left[2^{2N} \left(\frac{n}{16} \right)^N \right] \sim s_n \, N$$

- unusual correlations

6-vertex model:
$\sigma_i = \pm 1 \Leftrightarrow$ flux from A to B (B to A)
$\Rightarrow$ divergenceless condition and dipolar correlations [Isakov PRL 2004]

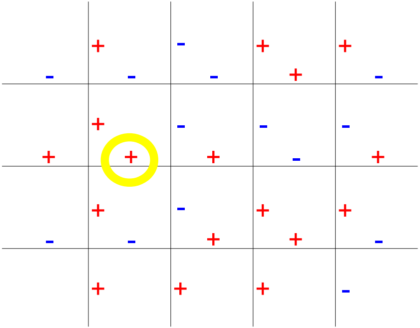
8-vertex model:
plaquette flips preserve minimal energy
$\Rightarrow$ zero-range corr. $\langle \sigma_i \sigma_j \rangle = 0, \forall i \neq j$ but topological properties

Excitations in the 8-vertex model

spin flip on ground state

$\Downarrow$

two defects: $\prod_{i \in s} \sigma_i = -1$



- spins next to defect flip at no energy cost (hop or annihilate)
- trivially deconfined
- elementary excitations are single defective sites
- lattice gas of (RW) particles that pair create or annihilate

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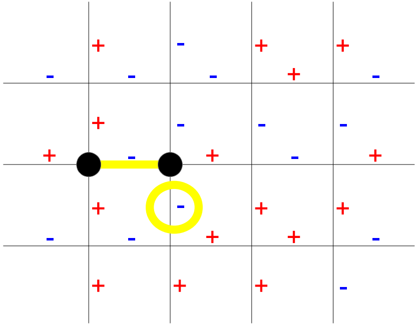

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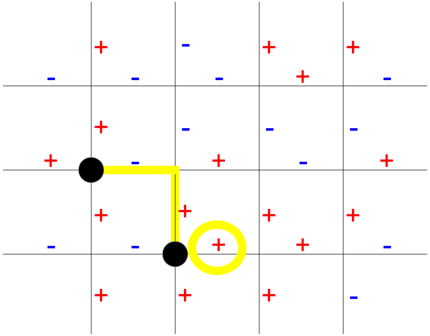

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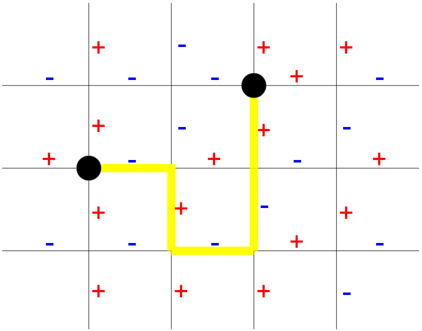

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Excitations in the 6-vertex model

spin flip on ground state

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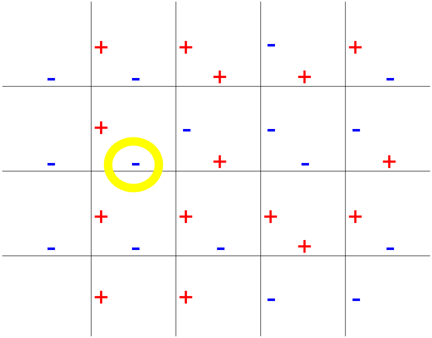
two defects: $\sum_{i \in s} \sigma_i = \pm 1$



- spins next to defect flip at no energy cost only along alternating sign paths $(+ - + - + - \ldots)$
- elementary excitations are single defective sites (deconfined)
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Excitations in the 6-vertex model

spin flip on ground state

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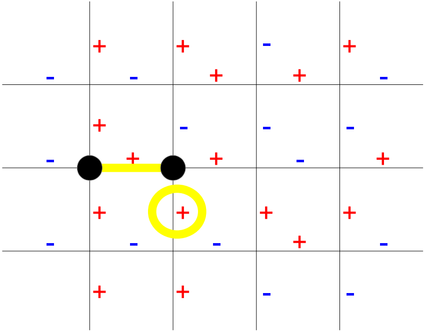

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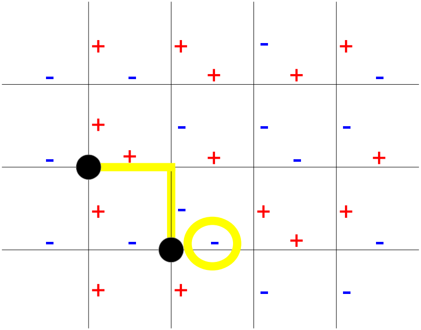

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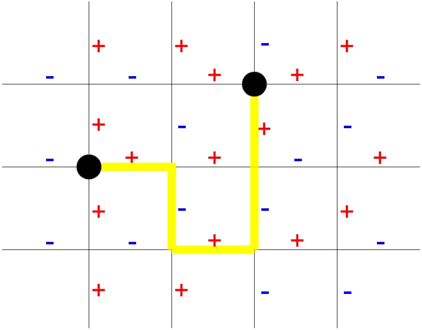

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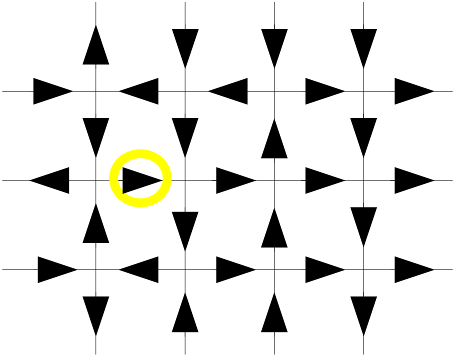
Excitations in the 6-vertex model (gauge flux rep.)

spin flip on ground state

⇓

pair of oppositely charged defects: **sinks** (3i1o) and **sources** (3o1i) of gauge flux



- defects move freely along oriented arrow paths
- close interplay: spins determine how defects move; defect motion rearranges spins
- lattice gas of gauge charges → spins mediate Coulomb int.

Excitations in the 6-vertex model (gauge flux rep.)

spin flip on ground state

$\Downarrow$

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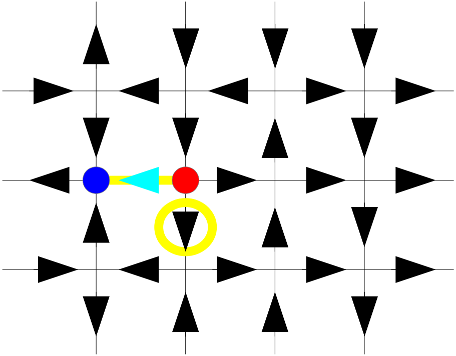

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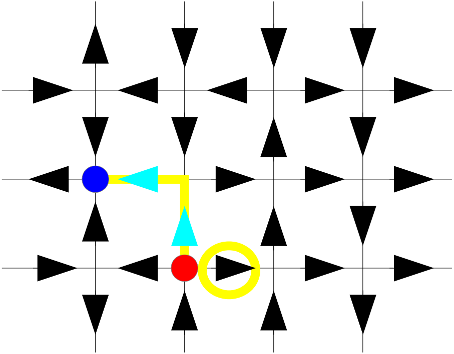

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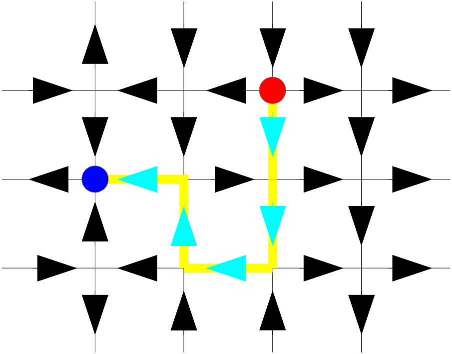

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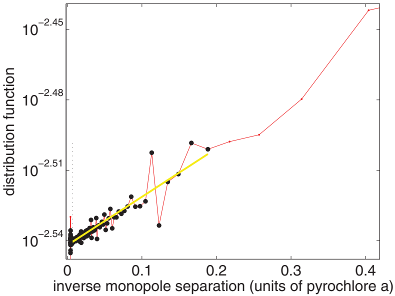
Parenthesis: entropic Coulomb interaction

- no energetic interactions between defects
- yet probability $\mathcal{P}(R)$ of two oppositely charged defects R apart $\sim \exp[C_d(R)]$ with Coulomb potential $C_d(R)$ in d dim.
- $\Rightarrow$ entropic Coulomb interaction $-T\, C_d(R)$



$d = 3,\ C_d(R) \sim 1/R$ CC et al. PRB 2011

Low temperature (classical) dynamics

behaviour controlled by sparse defect motion:

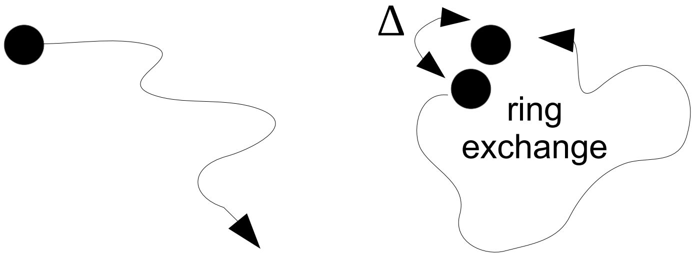
- 8-vertex: 2D random walk + pair creation/annihilation events (aka *reaction-diffusion* process)
- 6-vertex: constrained lattice gas motion + entropic Coulomb interactions

Toussaint et al. J. Chem. Phys. 1983
Ginzburg et al. PRE 1997
Ryzhkin et al. JETP 2005, EPL 2013
CC et al. PRL 2010, PRB 2019

Quantum spin liquids

$$H = H_\Delta + H_\delta \quad \text{where} \quad H_\delta \sim \cancel{H_{\text{defect int.}}} + H_{\text{defect hopping}}$$

- neglect $H_{\text{defect int.}}$ for simplicity
- hopping $t \lesssim \Delta \quad \to \quad$ defect dynamics (first order) + 'ground state' dynamics (perturbatively: $\overline{\Delta} \sim t\,(t/\Delta)^n$)



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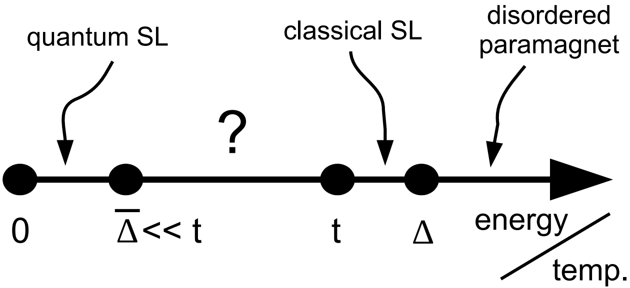
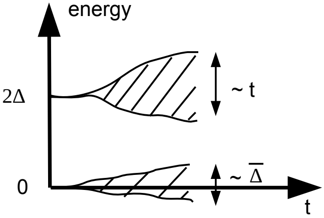




two-defect sector

zero-defect sector

Intermediate regime ($\overline{\Delta} \lesssim T < t$)

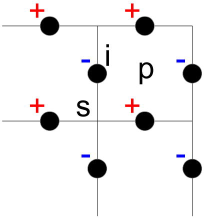
- intermediate between classical and quantum behaviour
- highest temperature where precursor QSL behaviour may appear ($\rightarrow$ experiments)
- underlying spins act as self-generated disorder in defect motion $\rightarrow$ localisation
- general framework hitherto unavailable...

(but interesting case studies)

arXiv:1909.08633
arXiv:1911.06331
arXiv:1911.05742
arXiv:2005.03036

$\rightarrow$ next talk (Thu 25$^{\text{th}}$ June, 16:30)

Quantum 8-vertex model (aka toric code in a field)



$$H_\Delta = -\Delta \sum_s \prod_{i\in s} \sigma_i^z$$

$$H_t = -t \sum_i \sigma_i^x$$

$$\Rightarrow \quad H_{\overline{\Delta}} = -\overline{\Delta} \sum_p \prod_{i\in p} \sigma_i^x , \quad \overline{\Delta} \sim \frac{t^4}{\Delta^3}$$

$H_\Delta + H_{\overline{\Delta}} =$ toric code

Kitaev 2003

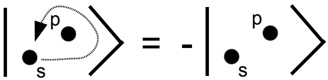
- H_Δ favours $\prod_{i\in s} \sigma_i^z = +1$
- $H_{\overline{\Delta}}$ favours $\prod_{i\in p} \sigma_i^x = +1$
- they commute and can be simultaneously satisfied

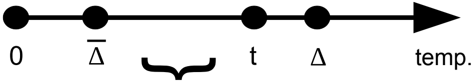
Quantum 8-vertex model (aka toric code in a field)

elementary excitations:

- star defects $\left(\prod_{i\in s}\sigma_i^z = -1, \text{ cost} \sim \Delta\right)$
- plaquette defects $\left(\prod_{i\in p}\sigma_i^x = -1, \text{ cost} \sim \overline{\Delta}\right)$

point-like, deconfined bosons, with
mutual semionic statistics



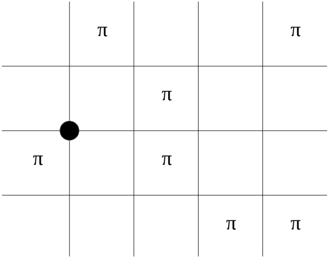


- star defects: sparse and hop coherently
- plaquette defects: thermally populated (dense and incoherent)

Quantum 8-vertex model ($\overline{\Delta} \lesssim T < t$)

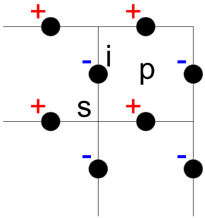
incoherent superposition of plaquette defects (ensemble average)

$+$ coherent star defect hopping



tight-binding charges in a random π-flux background:
Anderson localisation of emergent particles
($+$ thermodynamic response due to mutual statistics)

Quantum 6-vertex model (aka quantum spin ice in a field)



$$H_\Delta = -\Delta \sum_s \left(\sum_{i \in s} \sigma_i^z \right)^2$$

$$H_t = -t \sum_i \sigma_i^x$$

$$\Rightarrow \quad H_{\overline{\Delta}} = -\overline{\Delta} \sum_p \left(\sigma_1^+ \sigma_2^- \sigma_3^+ \sigma_4^- + \mathrm{h.c.} \right) ,$$

$$(\overline{\Delta} \sim t^4/\Delta^3)$$

$H_\Delta + H_{\overline{\Delta}}$ = quantum (square) spin ice

Hermele et al. 2004

- H_Δ favours $\sum_{i \in s} \sigma_i^z = 0$
- $H_{\overline{\Delta}}$ favours $+ - + -$ ('flippable') plaquettes
- they **do not** commute and cannot be simultaneously satisfied

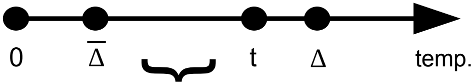
Quantum 6-vertex model (aka quantum spin ice in a field)

elementary excitations:

- star defects $(\sum_{i \in s} \sigma_i^z = \pm 1$, cost $\sim \Delta$, gauge charge $\pm 1)$
- plaquette dynamics promotes gauge symm. to QED
- plaq. defects: dual charges (cost $\overline{\Delta}$, not trivially related to $H_{\overline{\Delta}}$)
- gapless photons

Hermele et al. 2004

point-like, deconfined quasiparticles, with electromag. interactions
(and not immediately obvious statistics)

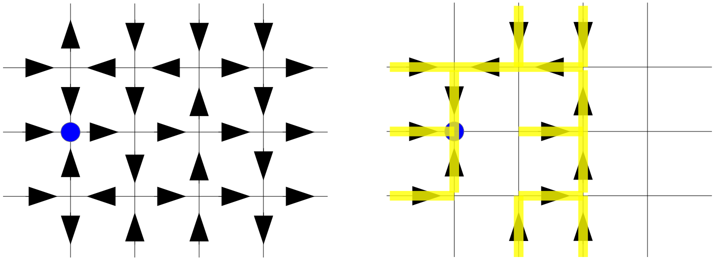


- star defects: sparse and hop coherently
- dual charges and photons: thermally populated

Quantum 6-vertex model ($\overline{\Delta} \lesssim T < t$)

working assumption: incoherent superposition of underlying spins
(ensemble average)

+ coherent star defect hopping



constrained dynamics $\leftrightarrow$ tight-binding on a random network $\leftrightarrow$
(emergent) configurational disorder

Conclusions

- frustration in magnetic systems opens a window into unusual and interesting spin liquid phases
- powerful effective modelling in terms of interplay between spin (GS) vacuum and quasiparticle excitations
  - classical: emergent symmetries, fractionalisation, reaction-diffusion processes and entropic interactions
  - quantum: dual quasiparticles and non-trivial statistics
- tease: interesting intermediate temperature regime with potential precursor signatures of QSL behaviour at lower temperatures