Extension Theory of Difference and Differential Operators

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Description of the research: Differential and difference expressions usually need to be equipped with boundary or initial conditions to describe physically relevant problems. Choosing these conditions means choosing an extension of an underlying minimal operator. For the Laplacian in a domain in R^n the most famous choices would be Dirichlet or Neumann conditions, for Sturm-Liouville problems on an interval one could also for example choose periodic boundary conditions. For Jacobi operators (coming from three-term recurrences), one needs to prescribe an initial condition and, in some cases, conditions at infinity.

This raises the question of which choices of conditions lead to particular properties of the extension, such as selfadjointness, dissipativity, sectoriality, properties of the spectrum etc.

There are many abstract ways of characterizing extensions of operators and their properties, e.g. [1-5], but very few applications to concrete operators, e.g. [6-8].

The PhD project will explore some of these applications and connections between the different characterizations, the exact direction of the thesis being dependent on the interests of the candidate. In all cases, it will be very useful to carefully look at some key examples before trying to develop a more general theory. This will provide an ideal way for a PhD student to become familiar with the topic before developing in depth knowledge of the theory.

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