Mathematical Physics and Integrable Systems
Spring Term 2019

22 January 2019
Andy Hone (University of Kent): *Some integrable maps and their Hirota bilinear forms*

We consider a family of birational maps found by Demskoi, Tran, van der Kamp and Quispel, containing a free parameter, which was shown to have many first integrals (at least half as many as the dimension of the space) and conjectured to be Liouville integrable. By extending it to a family with two parameters, we succeeded in proving this conjecture. The proof required several ingredients, including an initial heuristic analysis of singularities, the introduction of tau functions and Hirota bilinear equations, some associated cluster algebras and Poisson structures, and discrete KdV and Toda equations. The aim of this talk will be to given an overview of some of these ideas, including a neat arithmetical trick for guessing singularity patterns, and a way to prove the Laurent property and count degree growth in some cases that lie beyond cluster algebras. This is joint work with Theodoros Kouloukas and Reinout Quispel.

29 January 2019
Alexander Samokhin (Moscow): *On the Dubrovin conjecture for horospherical varieties of Picard rank one*

Given a Fano variety X, Dubrovin's conjecture is a statement that relates semisimplicity of the big quantum cohomology ring of X to the existence of a special structure on the derived category of coherent sheaves on X. One of the goals of our joint work with R.Gonzales, C.Pech, and N. Perrin is to establish Dubrovin's conjecture for the varieties in the title. In this talk, I will focus on the derived category side of Dubrovin's conjecture and try to explain how a refinement of that conjectural structure on the derived category naturally appears in the framework of non-commutative geometry. Despite all the technicalities above, I will assume no prior knowledge of derived categories of coherent sheaves and build my way to the case of our interest through numerous examples. This is based on https://arxiv.org/abs/1803.05063.

05 February 2019
Andy Hone (University of Kent): *Continued fractions and hyperelliptic curves*

Three-term linear recurrences are a central part of the theory of continued fractions, in the description of convergents, as well as being a feature of orthogonal polynomials. In this talk we consider some examples of discrete integrable systems associated with continued fraction expansions in hyperelliptic function fields. The case of elliptic curves corresponds to the well known example of the Somos-4 recurrence, and connects Hankel determinant formulae found by Chang, Hu & Xin with earlier work of van der Poorten. For the case of genus two and higher, new discrete integrable systems will be presented, with associated Hankel determinant formulae, and the connection with orthogonal polynomials will briefly be mentioned.

12 February 2019
Peter Clarkson (University of Kent): *Rational solutions of Painlevé equations*

In this talk I shall discuss rational solutions of Painleve equations. The six Painleve equations were derived by Painleve and his colleagues in the late 19th and early 20th centuries in a classification of second order equations whose solutions have no critical points. Although the general solutions of the six Painleve equations are transcendental, five of the Painleve equations possess rational solutions. These solutions are usually expressed in terms of logarithmic special polynomials that are Wronskians, often of classical orthogonal polynomials such as Hermite polynomials and Laguerre polynomials. It in known that the roots of these special polynomials are symmetric in the complex plane.I shall discuss some recent work on rational solutions of the third Painleve equation (with Law and Lin) and on rational solutions of the fourth Painleve hierarchy (with Gomez-Ullate, Grandati and Milson).

19 February 2019
Alfredo Deaño (University of Kent): *Critical behaviour in non-Hermitian random matrix ensembles and Painlevé transcendents*

Painlevé transcendents are special functions that appear in the theory of Hermitian random matrix ensembles of size NxN, both for finite N (for instance when studying gap probabilities for eigenvalues) and in the large N limit (when analysing critical transitions or the asymptotic behaviour of the partition function). In this talk we will explore similar results for non-Hermitian random matrix ensembles, in particular in the complex Ginibre ensemble with added algebraic singularities, where the eigenvalues are distributed in the complex plane.
Joint work with Nick J. Simm (University of Sussex, UK).

26 February 2019
Eduardo da Hora (Universidade Federal do Maranhão, Brazil): *First-order vortices inherent to the Abelian CP(2) theories*

I will present a series of recent developments which stablish the existence of first-order vortices coming from effective CP(2) theories endowed by Abelian gauge fields. In this talk, these results will be obtained by means of the Bogomol'nyi-Prasad-Sommerfield (BPS) prescription, i.e. via the minimization of the total energy of the effective model, from which I will obtain the corresponding first-order BPS equations and also the lower bound for the energy itself. One of the points to be highlighted during the presentation is the introduction of a differential constraint whose solution is the potential which engenders self-duality (i.e. the implementation of the BPS algorithm). I will show how the first-order formalism works when the dynamics of the gauge field is controlled by the Maxwell and the Chern-Simons terms, separately. The final solutions obey a particular set of boundary conditions which give rise to finite-energy configurations with no divergences. In this sense, another point to be discussed is that the BPS vortices coming from the Chern-Simons scenario can present both topological or nontopological profiles depending on the boundary conditions to be imposed in the asymptotic limit.

5 March 2019
Theodoros Kouloukas (University of Kent): *A new class of integrable Lotka-Volterra systems*

A parameter-dependent class of Hamiltonian (generalized) Lotka-Volterra systems is considered. We prove that this class contains Liouville integrable as well as superintegrable cases according to particular choices of the parameters. We determine sufficient conditions which ensure integrability and we investigate numerically the behavior of the system and the existence of possible extra integrals when these conditions are not satisfied.

12 March 2019
Anna Barbieri (University of Sheffield): *A Riemann-Hilbert problem for uncoupled BPS structures*

Riemann-Hilbert problems are inverse problems in the theory of differential equations, with a quite vast literature. On the other hand, BPS structures are discete data that locally describe the space of Bridgeland stability conditions of a CY3 category together with a generalised Donaldson-Thomas theory. After defining the notion of BPS structures I will introduce a Riemann-Hilbert problem naturally attached to BPS structures and present the solution in a simple case.

19 March 2019
Davide Proment (University of East Anglia): *Direct energy cascade in the two-dimensional Gross-Pitaesvkii model*

We analyse the nonlinear wave interactions and energy transfer in the two dimensional Gross-Pitaevskii equation (defocusing nonlinear Schroedinger equation in two spatial dimensions) using the wave turbulence (WT) framework. According to the WT theory applied to the Gross—Pitaevskii

equation, both the direct energy and inverse wave-action cascades are pathological and cannot take place in two spatial dimensions.
However, in the presence of a strong condensate, the dynamics changes switching from a 4-wave nonlinear interaction to a 3-wave one. In this regime, known as the Bogoliubov regime, the standard WT theory can still be applied: energy is the only conserved quantity and a direct energy cascade is expected.
After giving an introduction of the Gross—Pitaevskii equation used to model Bose—Einstein condensates and of the WT theory, I will present numerical results showing the existence of a direct energy cascade in this system. I will compare the numerical data with the theoretical WT predictions and discuss a possible experimental realisation of this numerical experiment. (joint work with U. Giuriato, J.C. Garreau, S. Nazarenko, and M. Onorato)

26 March 2019
Jacob Brooks (University of Surrey): *Existence of stationary fronts in a coupled system of two inhomogeneous sine-Gordon equations*

In this talk we investigate the existence of stationary fronts in a coupled system of two sine-Gordon equations with a smooth, "hat-like" spatial inhomogeneity. The spatial inhomogeneity corresponds to a spatially dependent scaling of the sine-Gordon potential term. Numerically, we find the uncoupled inhomogeneous sine-Gordon equation has stable stationary fronts. These front solutions persist in the coupled system. Carrying out further numerical investigation it is found that stable fronts bifurcate from these inhomogeneous sine-Gordon fronts provided the coupling between the two inhomogeneous sine-Gordon equations is strong enough. In order to analytically study the emerging fronts, we first approximate the smooth spatial inhomogeneity by a piecewise constant function. With this approximation, we prove analytically the existence of a pitchfork bifurcation of the inhomogeneous sine-Gordon fronts. To complete the argument, we use geometric singular perturbation theory to prove transverse fronts for a piecewise constant inhomogeneity persist for the smooth "hat-like" spatial inhomogeneity.

2 April 2019
Sylvain Carpentier (Columbia University): *The role of PreHamiltonian difference operators in (classical) integrable systems*

We discuss a relatively new algebraic structure in the theory of integrable systems (of differential-difference equations): the class of difference operators whose image is a sub Lie algebra of the algebra of evolutionary vector fields. These operators, called PreHamiltonian, encode most attributes of integrability for a given system. We will explain how they provide a natural non skew-symmetric generalization of the Hamiltonian (local and non-local) formalism, and discuss what is their geometric nature. This is a joint work with J.P. Wang and A. Mikhailov.

Mathematical Physics and Integrable Systems
Summer Term 2019

28 May 2019
Guido Carlet (Université de Bourgogne-Franche Comté, Dijon): *An infinite-dimensional Frobenius manifold structure on the space of simple analytic curves in the complex plane*

The notion of Frobenius manifold was introduced by B. Dubrovin as a geometric formulation of WDVV equations from two-dimensional topological field theory and proved important, in particular, in the study of integrable hierarchies of PDEs with one spatial dimension.
In this talk we introduce a structure of infinite-dimensional Frobenius manifold on a subspace in the space of pairs of functions analytic in the inner/outer regions of the unit circle in the complex plane with simple poles at 0/infinity respectively, which is related to integrable hierarchies with two spatial variables. From a joint work with B. Dubrovin and L. Ph. Mertens.

4 June 2019
Helen Christodoulidi (Academy of Athens): *The Fermi-Pasta-Ulam problem: Ergodic or not?*

The numerical experiment of Fermi, Pasta and Ulam (FPU) in 1954 aimed to probe ergodicity in an one-dimensional chain of N weakly nonlinearly coupled oscillators, however led to an unexpected integrable-like behaviour. It is noteworthy that FPU was the fi rst system which was solved numerically by a computer and it is linked with the birth of integrable systems through the discovery of solitons in the Korteweg-de Vries equation. In recent years there is a growing interest regarding the FPU model as perturbed Toda lattice, the latter of which is completely integrable. This idea goes back in a work of Flaschka, while recent studies suggest that the Toda integrals are the relevant dynamical observables for studying the FPU model. In the present talk I will discuss and compare the stages of dynamics in the FPU model for different classes of initial conditions, and propose a simple method to determine slow diffusion to estimate the two fundamental timescales, namely: i) the time of stability, where FPU behaves as Toda, and ii) the time to equilibrium.