Spectral Theory of Non-selfadjoint Operators

This is a proposal for a University PhD Studentship

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Description of the research: Spectral problems are often classified as 'selfadjoint' or 'non-selfadjoint' depending on whether they describe time-dependent systems which conserve or fail to conserve energy. Non-selfadjoint operators and associated spectral problems arise naturally in many physical application areas, such as hydrodynamics, magneto-hydrodynamics, lasers and nuclear scattering. The spectral behaviour of these operators is considerably more complex than that of selfadjoint operators. This leads to various new and unexpected phenomena which have implications for both the analysis and computation of spectral results. In particular, the spectrum is no longer necessarily confined to the real axis and the standard tools available for the selfadjoint case, such as the spectral theorem and variational principles, are no longer applicable.

In recent years there have been many developments in techniques for non-selfadjoint operators, including functional models for various classes of non-selfadjoint operators and the use of so-called boundary triples. Boundary triples allow the introduction of M-functions (Dirichlet-to-Neumann-type maps) for many operators. These are determined completely by boundary data, leading to interesting inverse problems, e.g. how much spectral information is contained in the M-function of a non-selfadjoint operator? Some first applications to partial differential equations have been considered by Brown, Marletta, Naboko and Wood, while connections to the extension theory of elliptic partial differential operators have been explored by Brown, Grubb and Wood. Using the techniques developed in these papers it will be possible to answer some outstanding questions in the extension theory of highly singular differential operators. Moreover, questions about the reality of the spectrum have applications in non-Hermitian quantum mechanics and in optics (e.g. PT-symmetric operators).

The PhD project will explore some of these questions, with the exact direction being dependent on the interests of the candidate. In all cases, it will be very useful to carefully look at some key examples before trying to develop a general theory. This will provide an ideal way for a PhD student to become familiar with the topic before developing in depth knowledge of the theory.

References:

1) Brown, B.M.; Grubb, G.; Wood, I.; M-functions for closed extensions of adjoint pairs of operators with applications to elliptic boundary problems. Math. Nachr., 3 (2009), 314 - 347.
2) Brown, B.M.; Marletta, M.; Naboko, S.; Wood, I.; Boundary triplets and M-functions for non-selfadjoint operators, with applications to elliptic PDEs and block operator matrices. J. LMS (2) 77 (2008), 700–718.