PhD Project in Mathematics

Polynomial modular rings of invariants of finite p-groups

Supervisor: Professor Peter Fleischmann, Co-supervisor: Dr R. James Shank

Description of Research Area:

The theory of groups and invariants is the mathematical language for analyzing symmetries. Objects or phenomena of interest and their properties are described mathematically in terms of solutions of systems of equations, involving numerical functions that depend on chosen coordinates. Changes of coordinates are then described by a suitable transformation group, acting on the system and its ingredients. Those functions which are unchanged by that group action, the `invariants', reveal the objective nature and the underlying symmetries of the studied phenomenon. It is therefore a major goal of invariant theory to provide general principles how to find all such invariants for a given group and how to perform efficient computations with them. Traditionally one looked at functions with real or complex coefficients, but more recent developments ask for invariants over more general coefficients, including modular fields. In that situation many of the "classical" results are unknown or known to be false, in which case one is looking for appropriate replacements. One major open question is addressed in this project: When is a modular ring of invariants a polynomial ring?

Description of project:

The last question is particularly interesting for finite p-groups in characteristic p. If the group action is linear, the existence of a polynomial invariant ring implies severe restrictions on the group structure, in the nonlinear case however, it has recently been shown in [3] that they always exist for any p-group. In both cases one is far away from a full classification, but certain patterns and conjectures arise. In this project the student will concentrate on the linear case and start by systematically constructing examples of rings of invariants for finite "transvection groups", exploiting some interesting structural properties, which have not yet been considered In that context ([4]). This investigation can start right away, on a relatively basic level; parallel to this the student will learn the necessary group theoretic and geometric background needed to generalize observations and prove corresponding theorems. Resources: Sufficient resources are in place (computers, software licences, literature, etc.)

Recent publications by the first supervisor connected to that work include:

[1] "The Noether Bound in Invariant Theory of Finite Groups", Adv. of Math. (156) (1) (2000), 23-32.

[2]: "Relative Invariants, Ideal Classes and Quasi-Canonical Modules of Modular Rings of Invariants", J. of Algebra 348, (2011), 110-134, (Fleischmann and Woodcock).

[3]: "Non-linear group actions with polynomial invariant rings and a structure theorem for modular Galois extensions", Proc. of LMS, 103(5), (2011),826-846, (Fleischmann and Woodcock).

[4] "On Finite Unipotent Transvection Groups and their Invariants", pg. 1-28, to appear in Quarterly Journal of Mathematics, Oxford, (Fleischmann)