Title: On multiple orthogonal polynomials.
PhD project in Mathematics at the University of Kent
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The classical orthogonal polynomials of Hermite, Laguerre, Bessel and Jacobi have been widely applied in many different areas of mathematics and physics. The concept has been extended to the so-called semiclassical orthogonal polynomials. The vast number of applications has made this subject very attractive and in the last decade there have been significant advances in the theory.

The proposal here is to work on the concept of multiple orthogonality, which is a very powerful extension of the concept of orthogonality. The multiple orthogonal polynomials have been widely and effectively applied in rational approximation, number theory (to prove irrationality and transcendence of certain constants), combinatorics, nonlinear discrete dynamical systems, operator theory, among others. They also became instrumental in random matrices and related stochastic processes, as they permit to describe the distribution of eigenvalues of the matrices. So, they are nowadays highly regarded and undergoing rapid development. There is a number of pertinent questions regarding the notions of "classical" or "semiclassical" in the context of multiple orthogonality. Most of these questions are intimately related to describing solutions of nonlinear differential and difference equations.

A range of research topics is available related to the modern theory of multiple orthogonal polynomials and includes applications to problems of current interest. Depending on the applicants' interests, the project may be more focused on the analysis of discrete or continuous multiple orthogonal polynomials and it may (or may not) involve Riemann-Hilbert problems, Painlevé equations, logarithmic potential theory, operator theory for difference operators. Applications to random matrix theory will be emphasised and developed throughout. The techniques involved appeal to a fruitful interplay between algebraic and analytical methods and they may be possibly flavored by combinatorial and integral transforms theory.

In addition, Kent will host the 14^{th} biannual international symposium on orthogonal polynomials and special functions and applications (OPSFA 14) as well as a summer school in the preceding week. These two events will be extremely beneficial for the potential candidate.

References

A. Aptekarev, M. Derevyagin, H. Miki and W. Van Assche, Multidimensional Toda Lattices: Continuous and Discrete Time, SIGMA 12 (2016), 054, 30 pp.

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M. Ismail, Quantum and Classical Orthogonal Polynomials, Encyclopedia of Mathematics and its Applications, vol. 98, Cambridge University Press, Cambridge, 2005.