Topological Solitons and their Moduli Spaces

This is a proposal for a University PhD Studentship

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Description of the research: Topological solitons are smooth, localised, finite energy solutions in non-linear field theories. The soliton number is conserved due to a topological constraint, such as a winding number or a non-trivial Chern class. Originally motivated from physics, these solitons give rise to interesting mathematical objects which can be studied using differential geometry and algebraic topology.

The PhD project mainly focusses on Abelian vortices on hyperbolic space as well as $\mathbb{C}P^1$ lumps and $\mathbb{R}P^2$ lumps where the domain is a general two dimensional manifold. Geometric properties of the soliton moduli space will be derived. For example, in some special cases, explicit formulas for metric and Ricci curvature can be calculated and studied. In other cases, global information such as the total volume, diameter and the total curvature can be computed. Furthermore, different types of dynamics of topological solitons will be studied. The most well-known dynamics is geodesic flow. Recently, a novel type of dynamics, known as Ricci magnetic geodesic motion, has been discovered.

The necessary computations will mainly be analytical. However, the resulting formulas can become lengthy so that the use of a suitable symbolic computer algebra package such as Maple is essential. Some numerical calculations may also become necessary.

References:

[1] N.S. Manton and P.M. Sutcliffe, Topological Solitons, Cambridge University Press, 2004.