PhD Project in Mathematics

Modular Invariants and Galois Algebras

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Description of Research Area:

The discovery of symmetry in nature is one of the most fundamental and universal intellectual achievements. The mathematical language for analyzing symmetries is the theory of groups and their invariants: objects or phenomena of interest and their properties are described mathematically in terms of solutions of systems of equations, involving numerical functions that depend on chosen coordinates. Changes of coordinates are then described by a suitable transformation group, acting on the system and its ingredients. Those functions which are unchanged by that group action, the `invariants', reveal the objective nature and the underlying symmetries of the studied phenomenon. It is therefore a major goal of invariant theory to provide general principles how to find all such invariants for a given group and how to perform efficient computations with them. Traditionally one looked at functions with real or complex coefficients, but more recent developments ask for invariants over more general coefficients, including modular fields. In that situation many of the "classical" results theory are unknown or known to be false, in which case one is looking for appropriate replacements. Key open questions include: How efficiently can such a ring be constructed? When is a modular ring of invariants a polynomial ring, or a Cohen-Macaulay ring? How do invariant rings behave under standard ring theoretical operations (e.g. localisation, completion etc.?)

Description of project:

In the series of recent papers [2, 4-6] a new branch of modular invariant theory has been established , which has close links to Galois theory and number theory. The trace-surjective algebras investigated in [4] provide models for "essential Galois fields" for p-groups over modular fields as well as techniques to explicitly compute these algebras for interesting classes of p-groups, like arbitrary cyclic groups or extra-special groups. There are several conjectures, e.g. on the "essential dimension" of groups, that hopefully can be proven using these techniques. This requires the solution of explicit cohomology equations, the solutions of which are known only in special cases. A second strand of that project deals with the generalization of the results in [2, 4-6] to arbitrary finite groups. Thus the project will involve experimental computation, starting early on, as well as theoretical analysis, which need some more preparation. Methodically, the research in this area is closely related to the areas of commutative algebra, group theory, Galois theory and representation theory.

Publications by the first supervisor connected to that work include:

[1] "On the Depth of Modular Invariant Rings for Groups Cp x Cp", Progress in Mathematics 278, Birkhaeuser, (2010), 45-61,(Elmer and Fleischmann),

[2] "Homomorphisms, Localizations and a New Algorithm to construct Invariant Rings of Finite Groups" J. of Algebra 309, (2007), 497-517, 2007 (Fleischmann, Kemper and Woodcock)

[3]: "Relative Invariants, Ideal Classes and Quasi-Canonical Modules of Modular Rings of Invariants", J. of Algebra 348, (2011), 110-134, (Fleischmann and Woodcock).

[4]: "Non-linear group actions with polynomial invariant rings and a structure theorem for modular Galois extensions,", Proc. of LMS, 103(5), (2011),826-846, (Fleischmann and Woodcock).

[5]: "Universal Galois algebras and cohomology of p-grouls" Journal of Pure and Applied Algebra, vol. 217, no. 3, pp. 530–545, 2013, (Fleischmann and Woodcock).

[6]: "Galois ring extensions and localized modular rings of invariants of p-groups", to appear in Transformation Groups, (Fleischmann and Woodcock).