

## Representation theory of symmetric groups, wreath products and related algebras.

The representation theory of symmetric groups has been studied for over a century. Although mathematicians have a good understanding of some parts (see, for example, [5]), fundamental questions remain unanswered. Representations of the wreath product of two symmetric groups sometimes arise when studying the representation theory of symmetric groups (for example in [1]). A survey of some of the known representation theory of wreath products, such as a description of their irreducible modules, is given in Chapter 4 of [5] and a more general approach can be found in [2].

In this project, you will address questions concerning representations of the wreath product of two symmetric groups and related representations of symmetric groups over arbitrary fields. You will build on this to study other algebras including certain diagram algebras (roughly speaking, algebras with a basis given by certain diagrams and a multiplication coming from concatenation of diagrams). The ideas from [3] and [4] will provide key techniques to do this. The necessary research will require good skill in algebra and aptitude in combinatorics, and may also benefit from computing skills.

### References:

1. J. Chuang, R. Kessar. Symmetric groups, wreath products, Morita equivalences, and Broué's abelian defect group conjecture. *Bull. London Math. Soc.*, 34 (2002), 174-184.
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3. J. Graham and G. Lehrer. Cellular algebras. *Invent. Math.*, 123 (1996), 1-34.
4. R. Hartmann, A. Henke, S. Koenig, R. Paget. Cohomological stratification of diagram algebras. *Math. Ann.*, 347 (2010), 765-804
5. G. James, A. Kerber, *The representation theory of the symmetric group*, Encyclopedia of Mathematics and its Applications, vol. 16, Addison-Wesley Publishing Co., Reading, Mass., 1981.