Title: Noncommutative Algebra, Poisson Geometry and Combinatorics

PhD project in Mathematics at the University of Kent

Supervisor: Dr Stéphane Launois

Background: The general topics of these PhD projects are Poisson algebras, their quantisations and beyond. These projects involve techniques from various areas: ring theory, representation theory, algebraic geometry, algebraic combinatorics, Lie theory, etc.

Poisson algebras are commutative algebras that first appeared in the work of Siméon-Denis Poisson two centuries ago when he was studying the three-body problem in celestial mechanics. Since then, Poisson algebras have shown to be connected to many areas of mathematics and physics.

One way to approach Poisson algebras is via quantisation. Roughly speaking, the idea is to use the Poisson bracket under consideration to deform the commutative product and obtain a noncommutative product suitable for quantum mechanics. To give the reader a flavour of this subject, consider the simple example of the quantum plane A = C□x; y□ generated by two indeterminates x and y subject to the relation $xy = qyx$. If q is different from 0,1, then this algebra is noncommutative. However, when $q = 1$, this algebra becomes the commutative polynomial ring P in two variables. Thus we think of A as a noncommutative deformation of P. Even better, one can show that the algebra A gives rise to a Poisson bracket on P by a semiclassicalisation process.

Recently, these ideas have led to much progress in the study of totally nonnegative matrices, that is, matrices whose minors are all nonnegative. This class of matrices has recently been used in areas as diverse as computer science, chemistry, physics and economics. In the 90’s Lusztig generalised this notion and defined the space of totally nonnegative elements in a real flag variety—a very beautiful geometric object from algebraic Lie theory. As often, putting things in a more general context has led to many ground breaking developments such as for instance the theory of cluster algebras by Fomin and Zelevinsky.

Indicative PhD projects: Depending on the interests of applicants, I can offer projects around the following topics:

• Study the semiclassicalisation process using tools from noncommutative algebra and Poisson geometry.

• Application of noncommutative algebra techniques to the study of totally nonnegative matrices (and beyond).

• Study of quantum cluster algebras.

Bibliography: I am only listing three articles that are representative of my current research interests.

- S. Launois and T.H. Lenagan, *From totally nonnegative matrices to quantum matrices and back, via Poisson geometry*, to appear in the Proceedings of the Belfast Workshop on Algebra, Combinatorics and Dynamics 2009. arXiv:0911.2990
- J.E. Grabowski and S. Launois, *Graded quantum cluster algebras and an application to quantum Grassmannians*, to appear in Proceedings of the London Mathematical Society. arXiv:1301.2133
- J.P. Bell, S. Launois, O. Leon Sanchez and R. Moosa, *Poisson algebras via model theory and differential-algebraic geometry*, arXiv:1406.0867